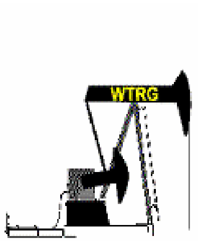
“Modelación de flujos multifasicos en yacimientos de petróleo”

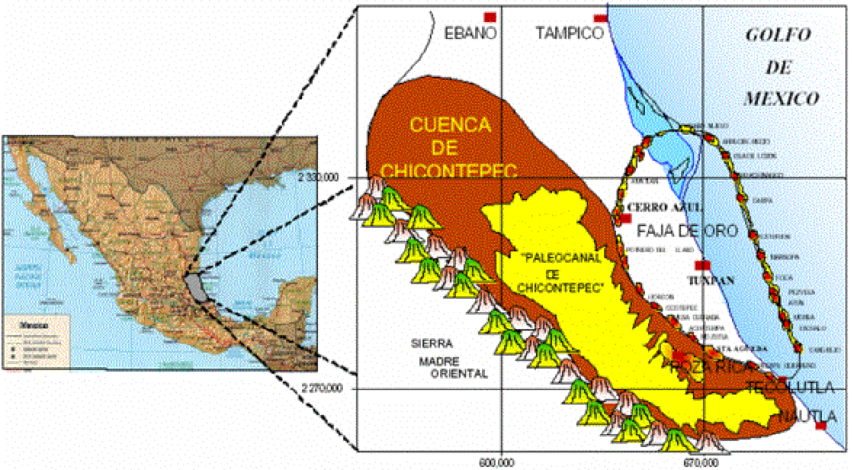
Dr. Alexander Balankin

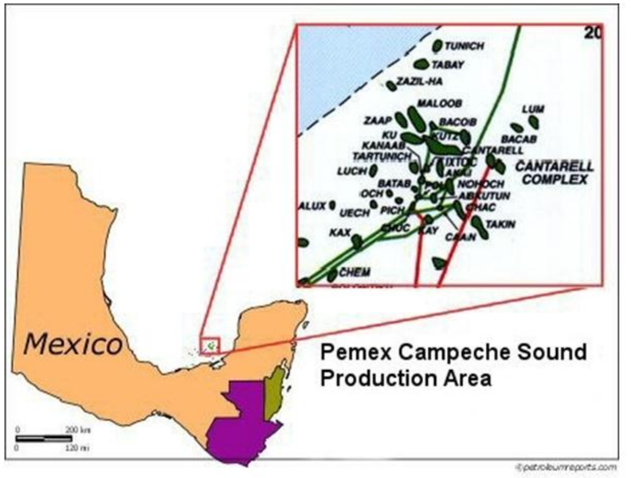


Proyecto: SENER-CONACyT-Hydrocarburos **143927**

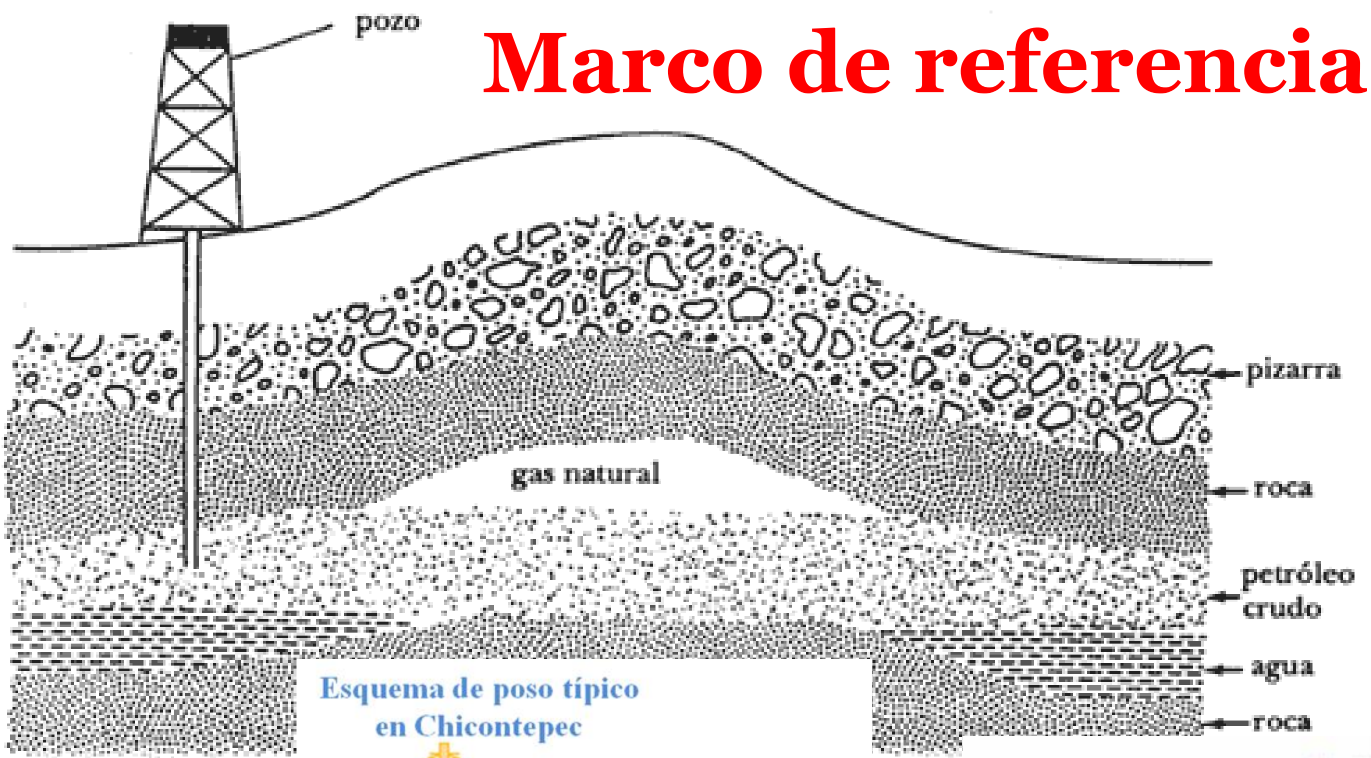
Modelos Fractales para Caracterización de Yacimientos Sistemas Heterogéneos de Difusión Lenta

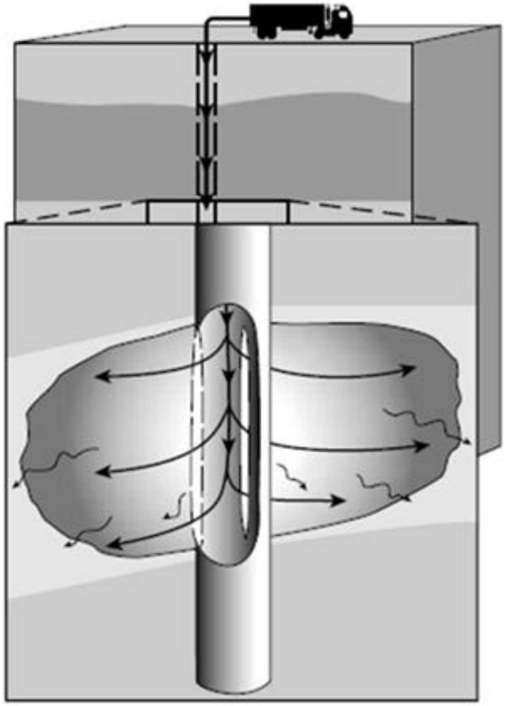
SEPI-ESIME-Zacatenco, Programa - Ingeniería de Sistemas



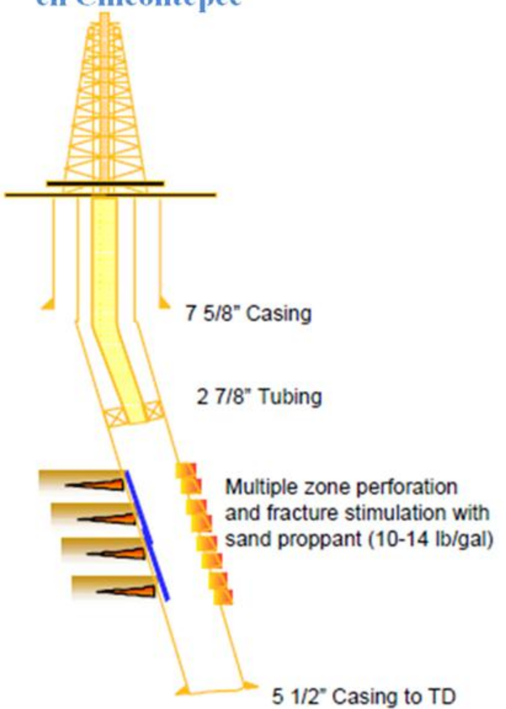


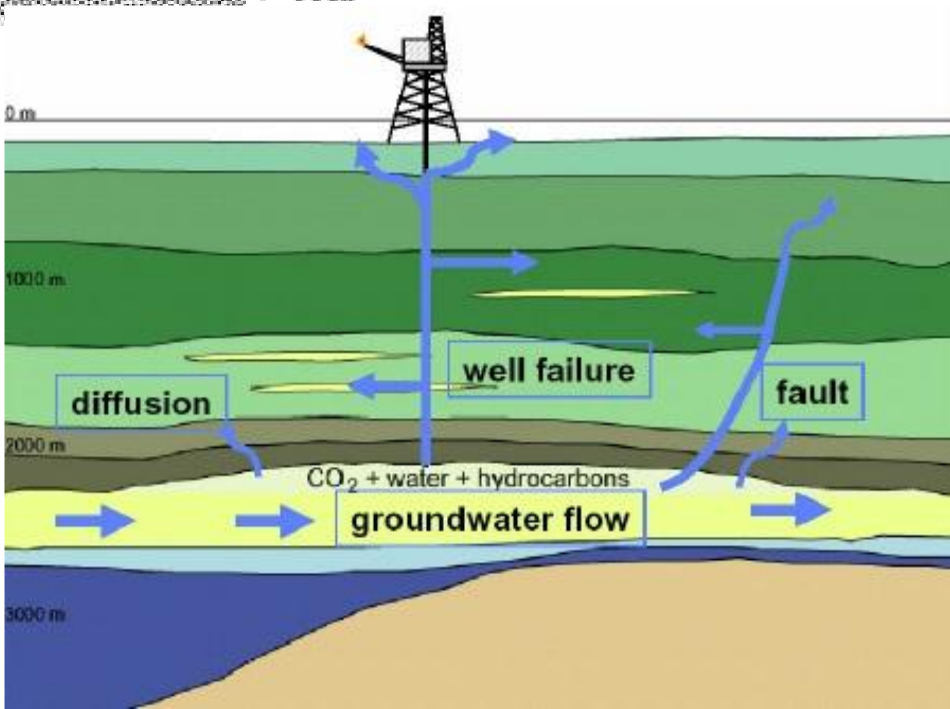
Marco de referencia





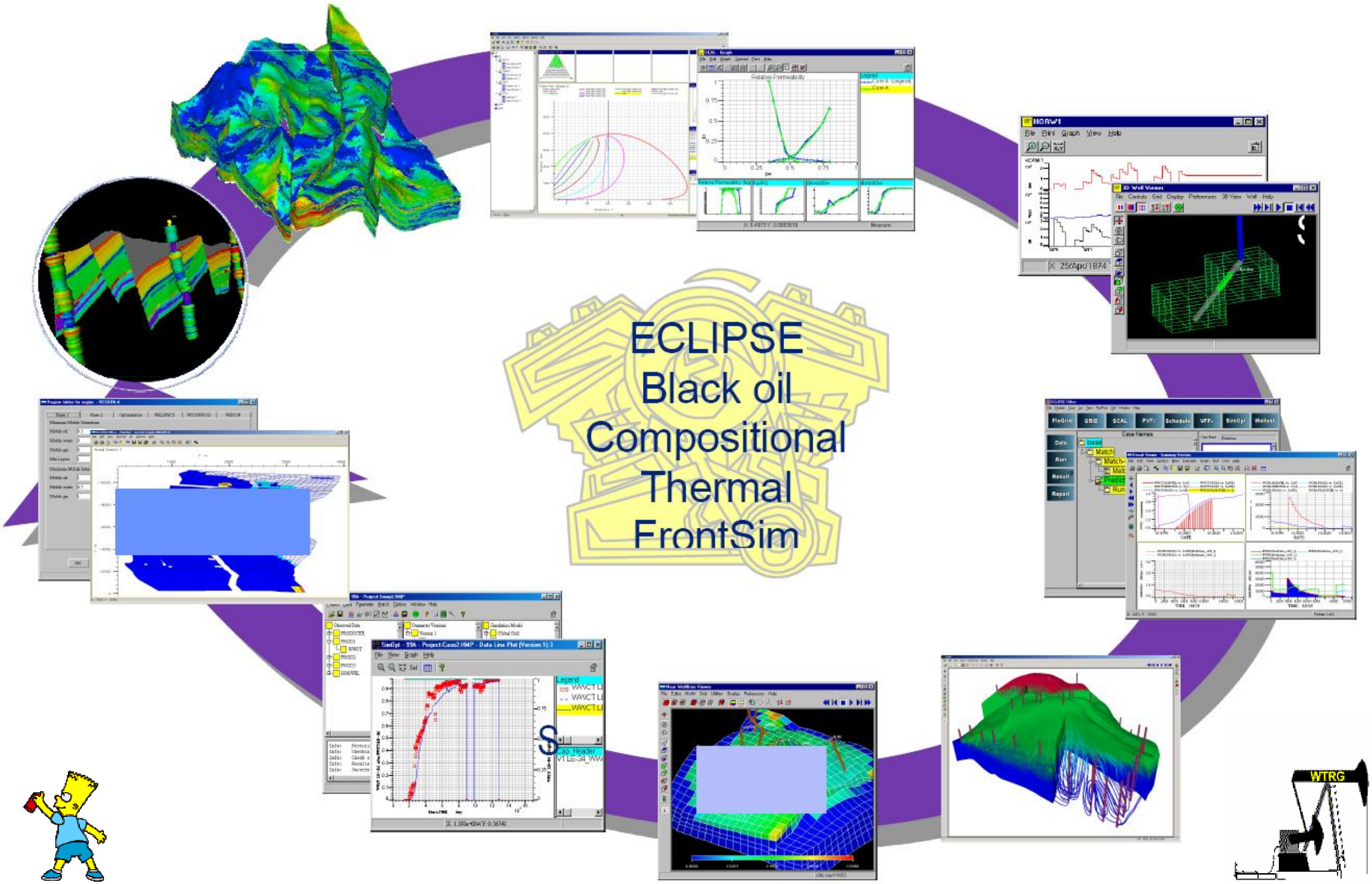
Esquema de poso típico en Chicontepec





Software ECLIPCE

Geometría – *Euclidiana* **Método – *Diferencias Finitas***



Marco de referencia

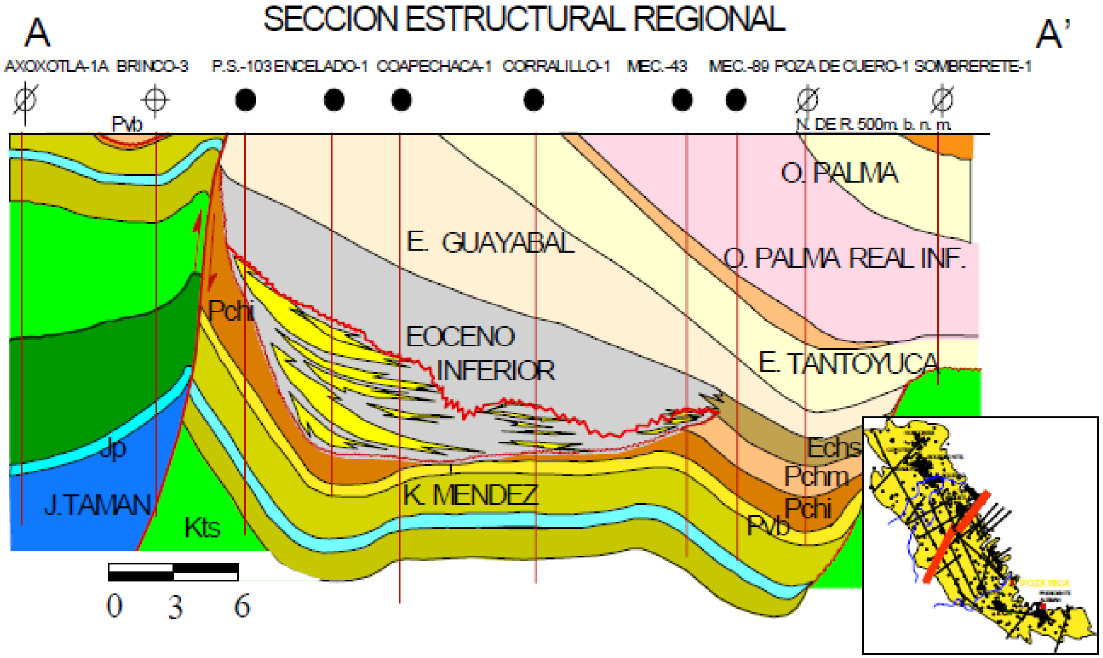
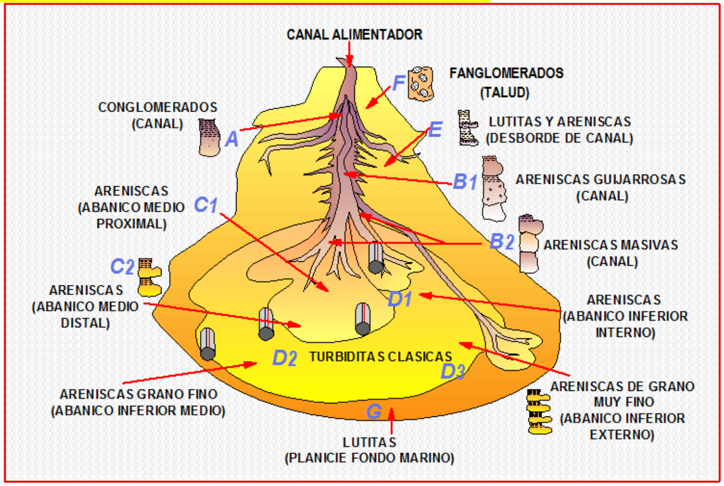


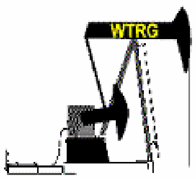
Fig. 13 Chicontepec basin regional cross-section (Pemex).



- **Modelo Sedimentológico conceptual de *Abanico Submarino de Walker,* definido mediante la descripción de 390 núcleos de 50 pozos**

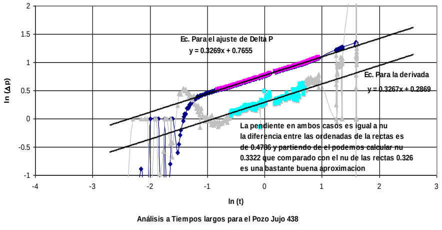
- Alta estratificación areno-arcillosa con bajos valores de permeabilidad

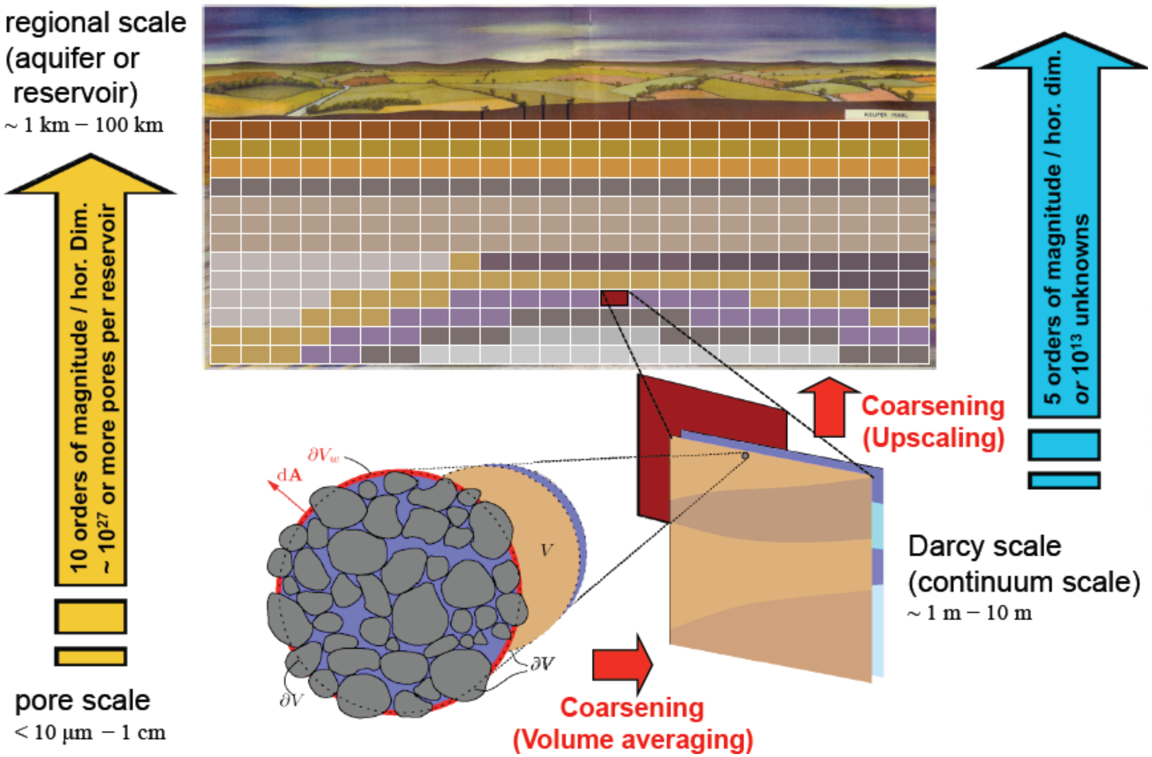
Fracturamiento hidráulico es indispensable

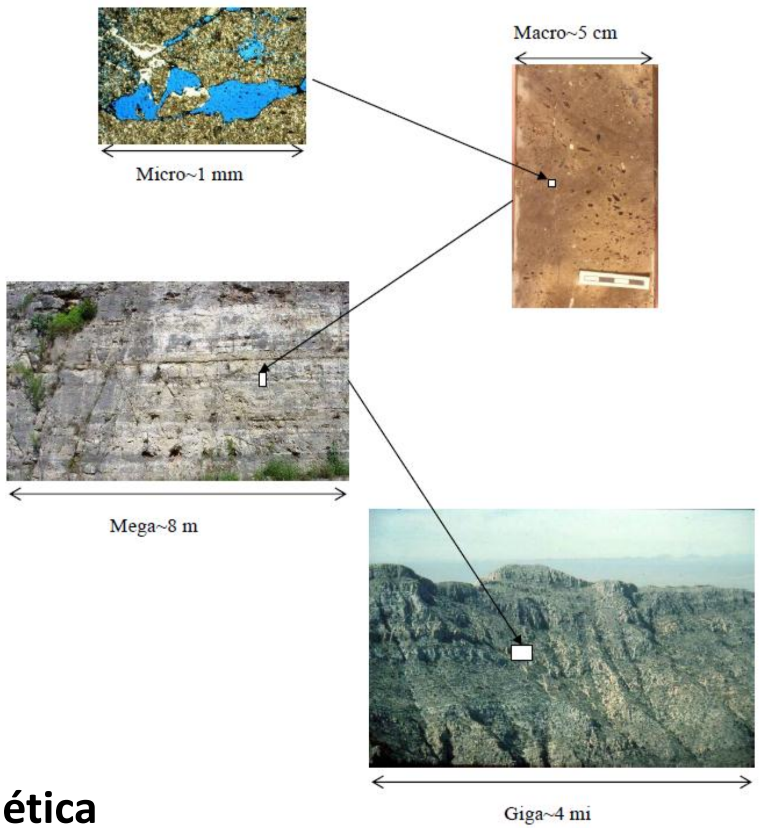


- Gran discontinuidad a los cuerpos arenosos: Distribución espacial y vertical irr

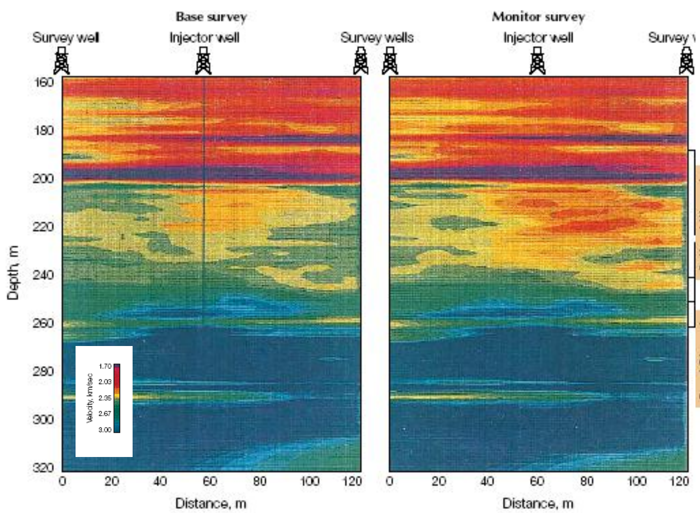
Geometría Fractal



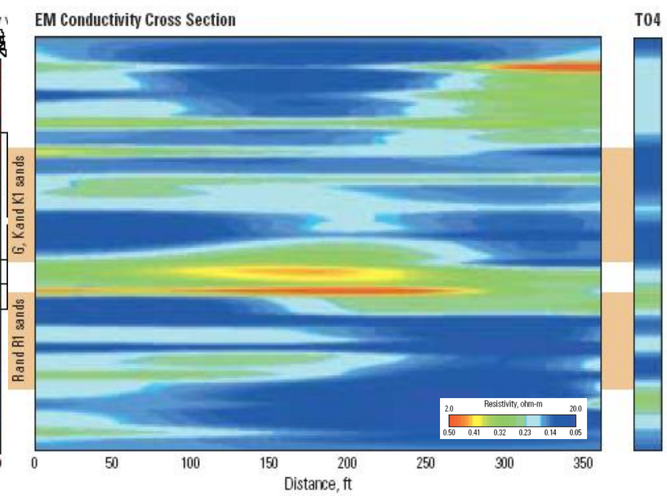




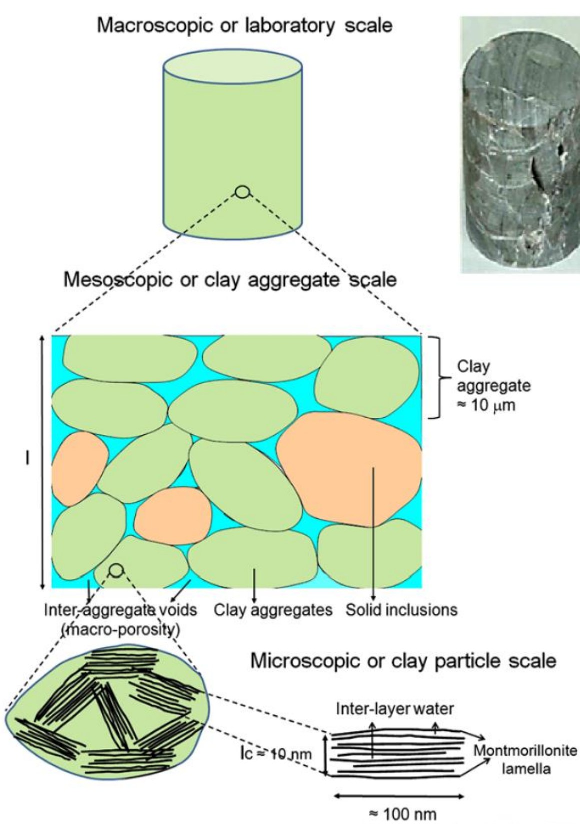
Sísmica

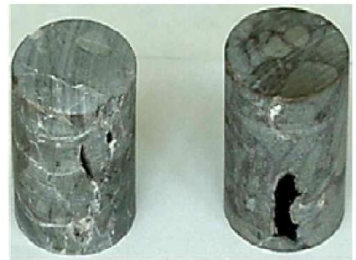


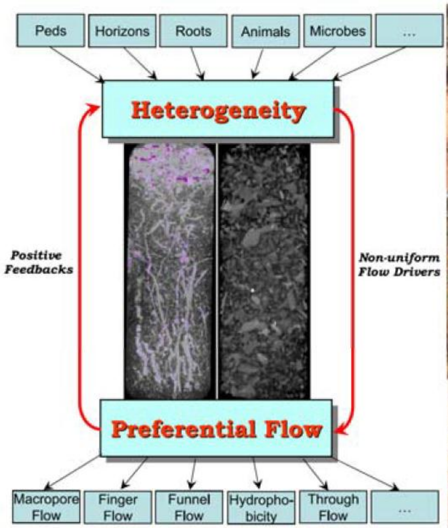
Electromagnética

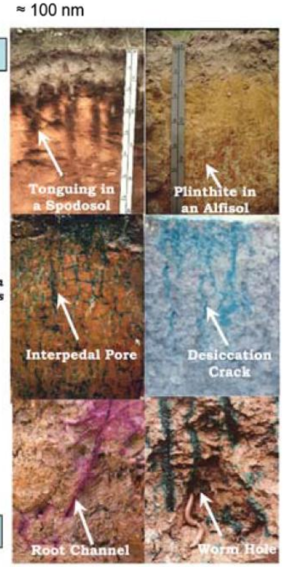


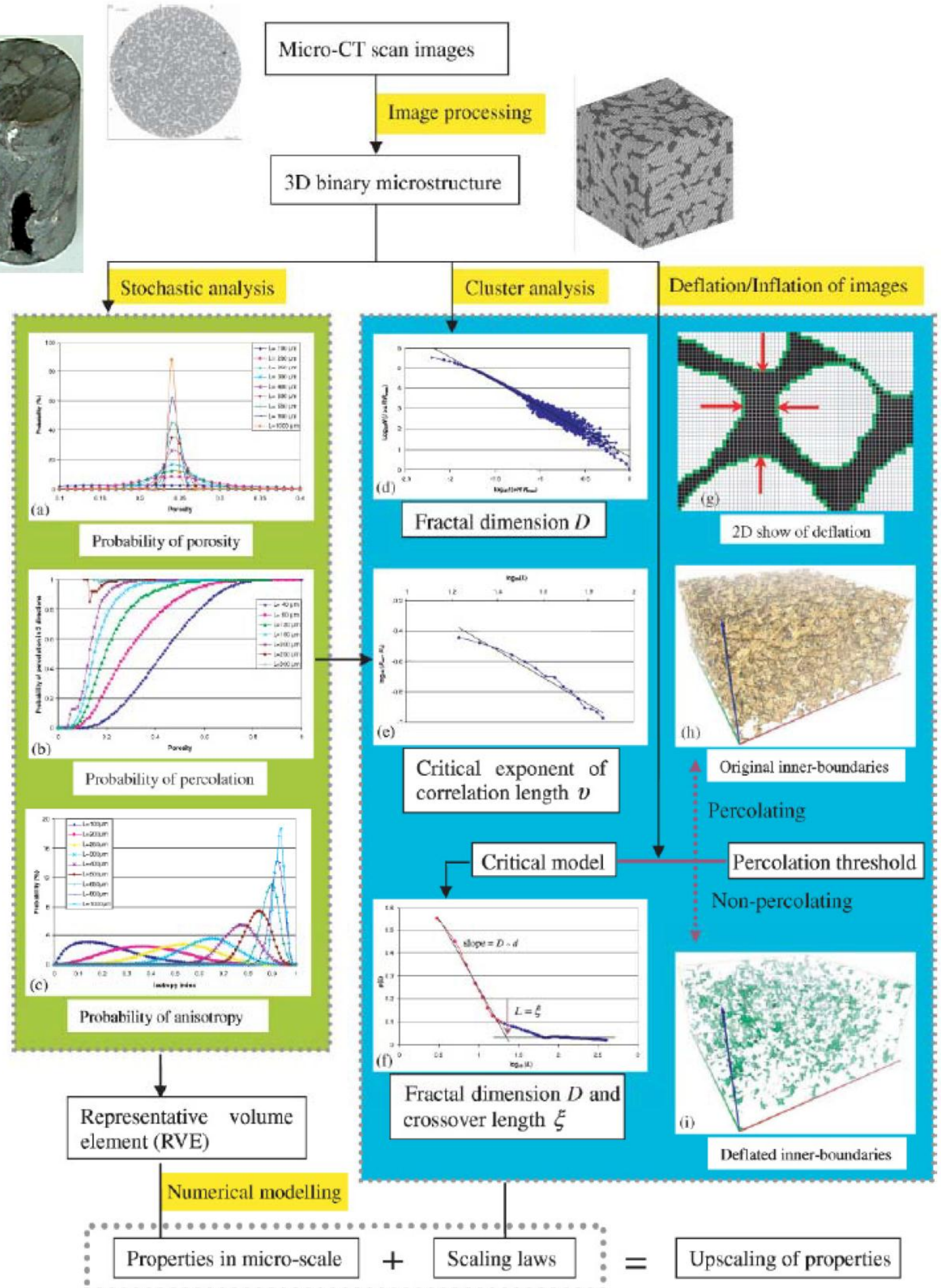
Núcleos


Macroscopic or laboratory scale
Mesoscopic or clay aggregate scale
Clay aggregate ≈ 10 μm
l
Inter-aggregate voids (macro-porosity)
Clay aggregates
Solid inclusions
Microscopic or clay particle scale
Inter-layer water
lc ≈ 10 nm
Montmorillonite lamella
≈ 100 nm




Peds
Horizons
Roots
Animals
Microbes
...
Heterogeneity
Positive Feedbacks
Non-uniform Flow Drivers
Preferential Flow
Macropore Flow
Finger Flow
Funnel Flow
Hydrophobicity
Through Flow
...

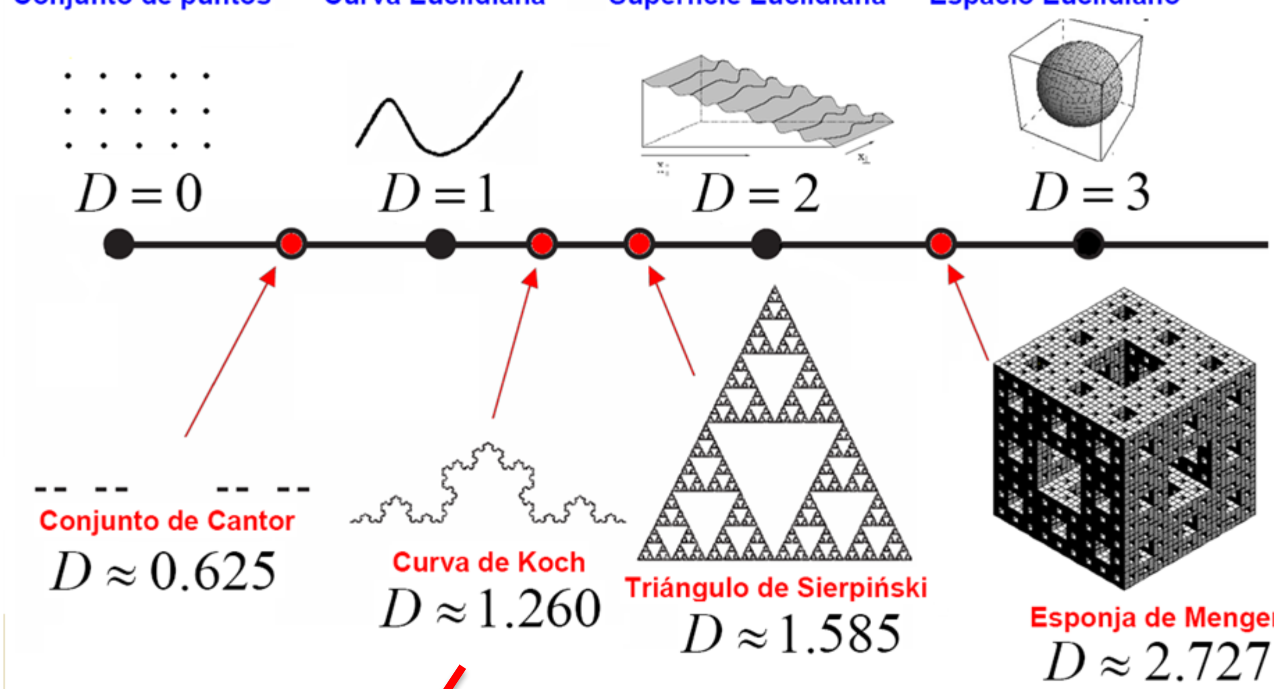

Tonguing in a Spodosol
Plinthite in an Alfisol
Interpedal Pore
Desiccation Crack
Root Channel
Worm Hole


Micro-CT scan images
Image processing
3D binary microstructure
Stochastic analysis
Cluster analysis
Deflation/Inflation of images
Probability of porosity
Probability of percolation
Probability of anisotropy
Fractal dimension D
Critical exponent of correlation length υ
Critical model
Percolation threshold
Percolating
Non-percolating
slope = D - d
Fractal dimension D and crossover length ξ
2D show of deflation
Original inner-boundaries
Deflated inner-boundaries
Representative volume element (RVE)
Numerical modelling
Properties in micro-scale
+
Scaling laws
=
Upscaling of properties

Formalización del concepto de Fractal

La geometría fractal permite estudiar fenómenos irregulares que no pueden ser caracterizados con las teorías geométricas clásicas.

La geometría tradicional (euclídea) se encarga de las propiedades y de las mediciones de objetos tales como **puntos**, **líneas**, **planos** y **volúmenes**.



Invarianza a cambios de escala. Misma estructura (determinsta o estadística) a cualquier escala.

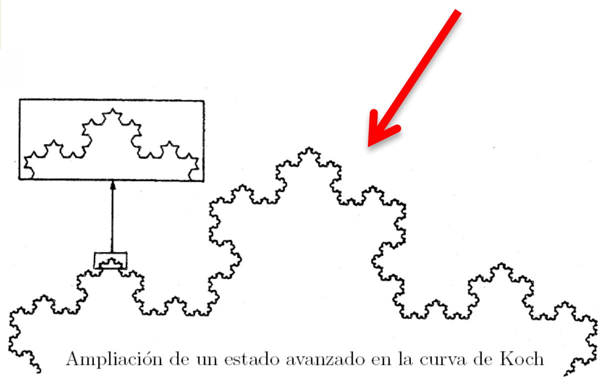
Leyes de potencia.

$$m(l) = l^D$$

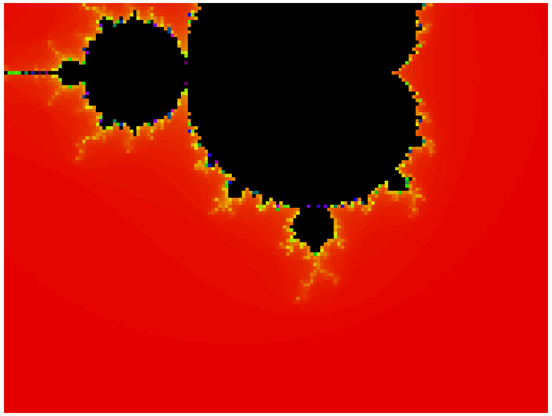
Soluciones de EDs

Curvas y superficies de interpolación

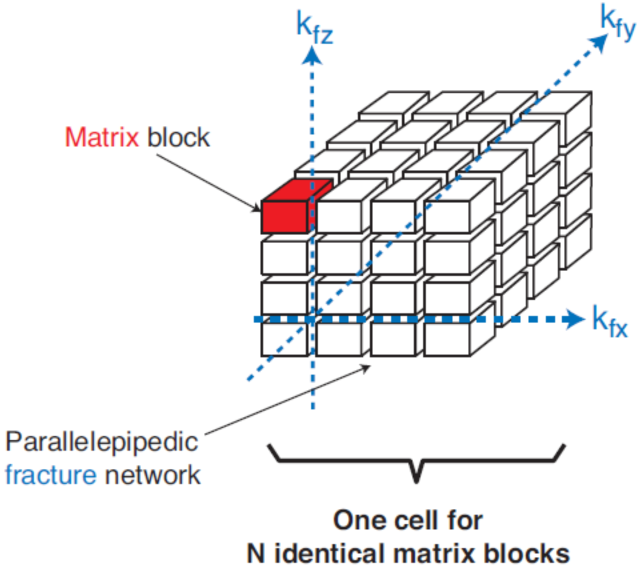
Sistemas de Funciones Iteradas

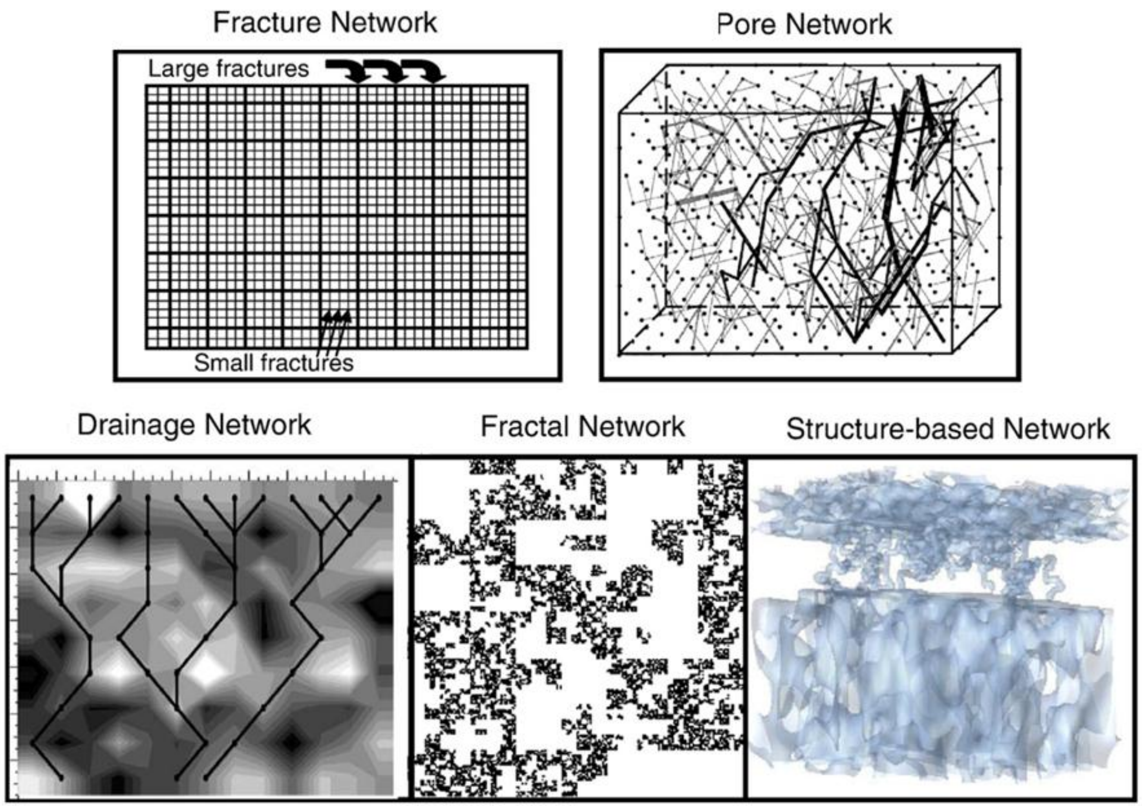


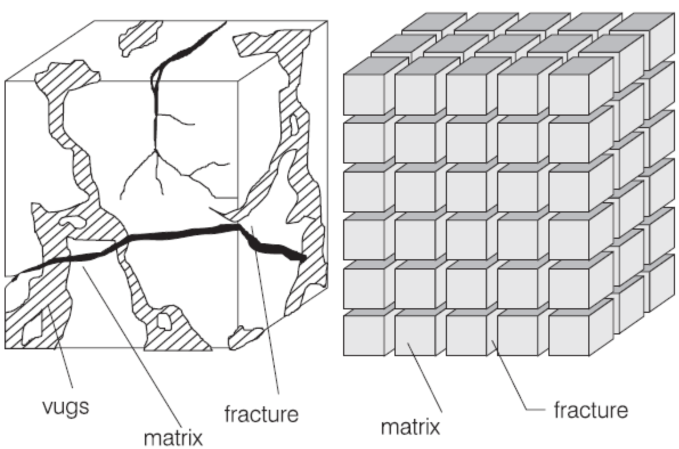
Ampliación de un estado avanzado en la curva de Koch

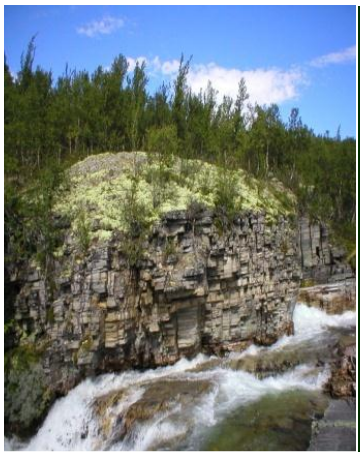


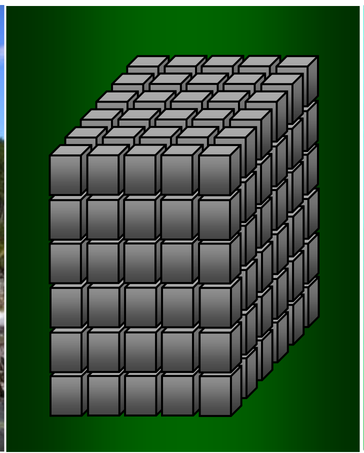
Modelos conceptuales de redes de fracturas

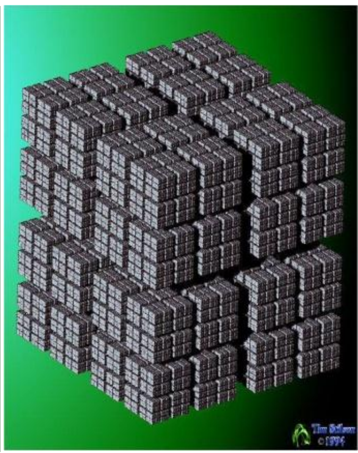




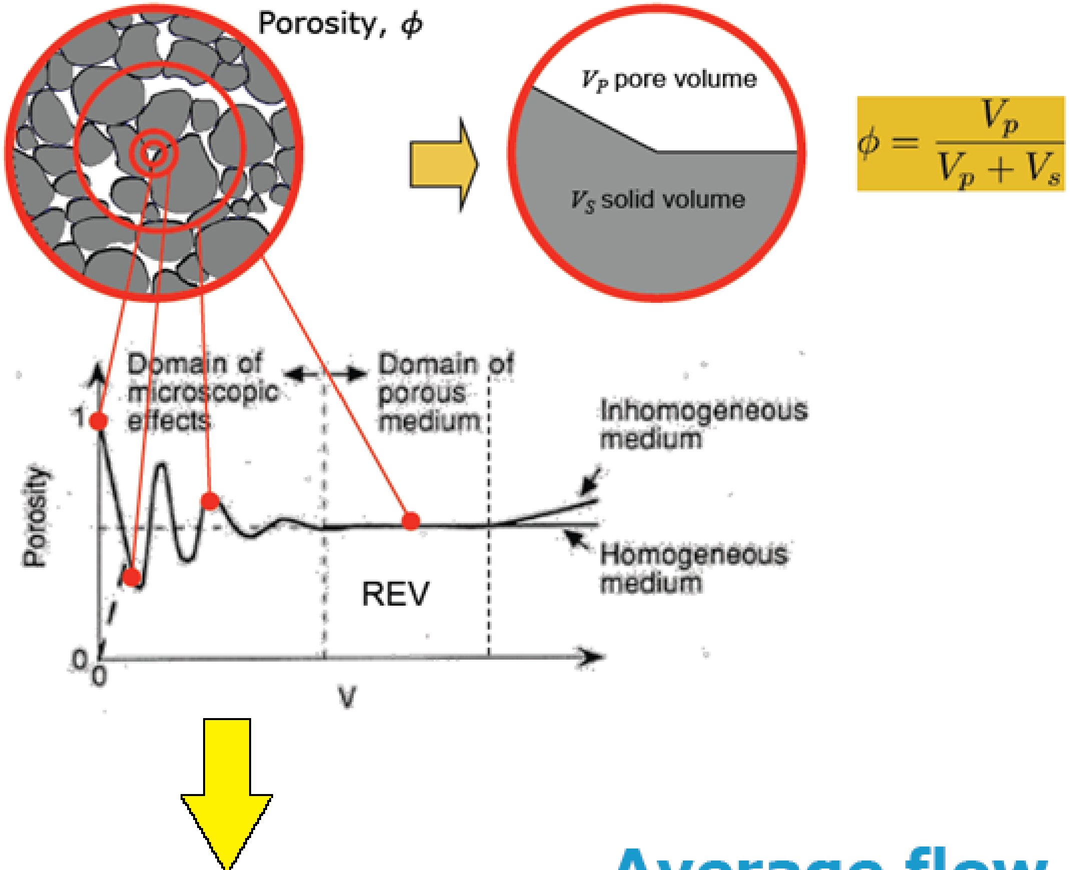


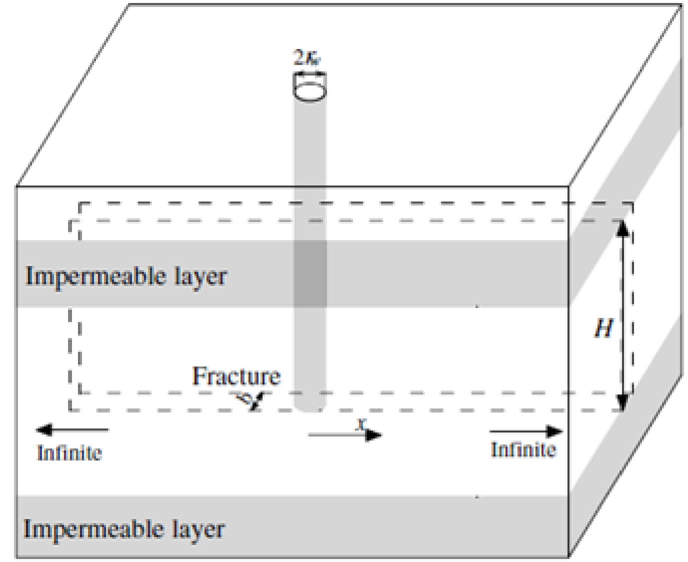


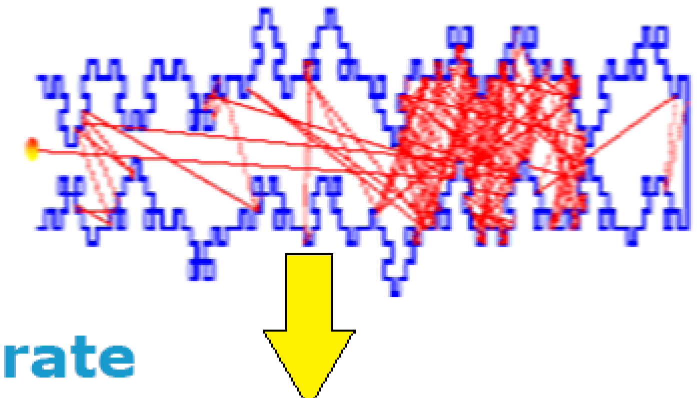




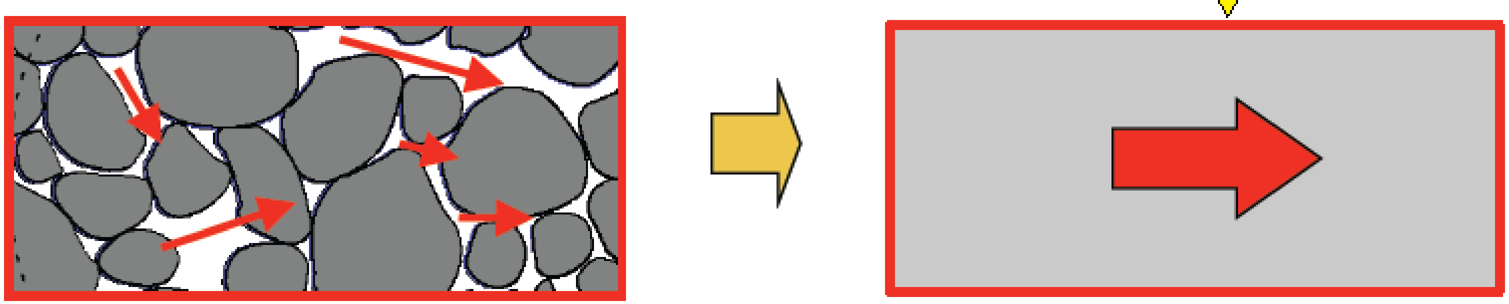
Representative Elementary Volume (REV)





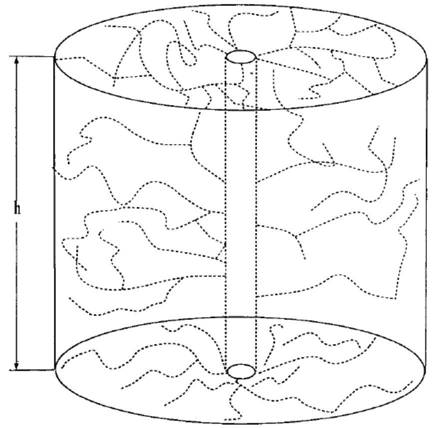


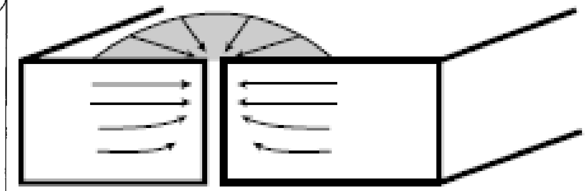
Average flow rate



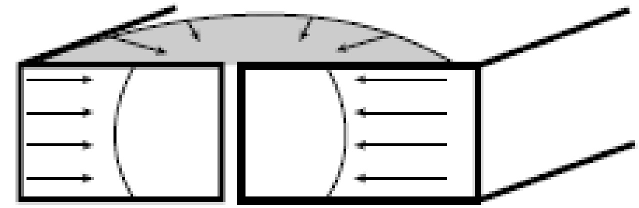
Estado de Arte

Flujo Radial:





Complete radial flow



Pseudoradial flow to fracture

Euclidean

$$\frac{\partial p_D}{\partial t_D} = \frac{1}{r_D^{n-1}} \frac{\partial}{\partial r_D}\left(r_D^{n-1} \frac{\partial p_D}{\partial r_D} \right)$$

Barker (1981)

$$\frac{\partial p_D}{\partial t_D} = \frac{1}{r_D^{D^*-1}} \frac{\partial}{\partial r_D}\left(r_D^{D^*-1} \frac{\partial p_D}{\partial r_D} \right) = \frac{\partial^2 p_D}{\partial r_D^2} + \frac{D^* - 1}{r_D} \frac{\partial p_D}{\partial r_D}$$

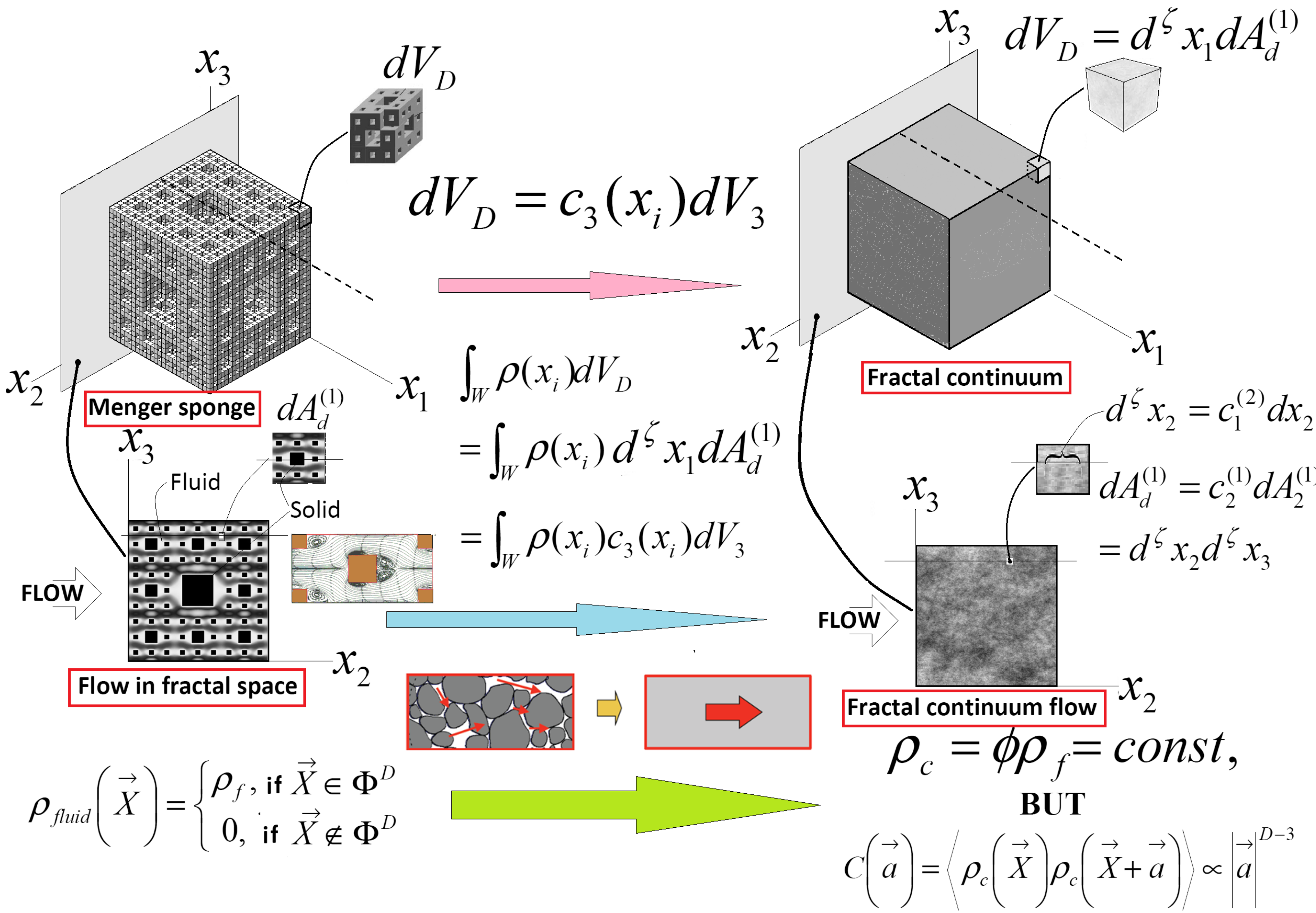
Chang-Yortsos (1990)

$$\frac{\partial p_D}{\partial t_D} = \frac{1}{r_D^{D_n-1}} \frac{\partial}{\partial r_D}\left(r_D^{D_n-1-\theta} \frac{\partial p_D}{\partial r_D} \right) = \frac{1}{r_D^{\theta}}\left(\frac{\partial^2 p}{\partial r_D^2} + \frac{D_n - 1 - \theta}{r_D} \frac{\partial p_D}{\partial r_D} \right)$$

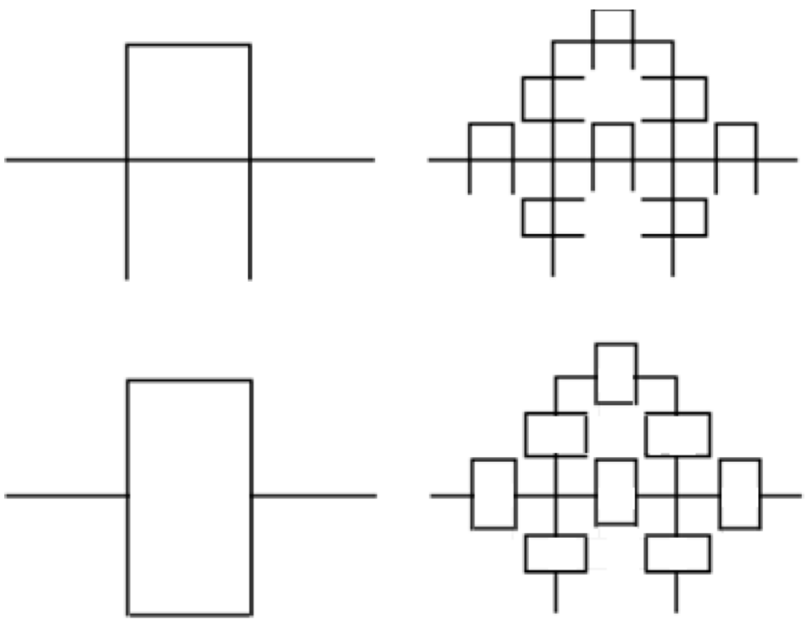
Metzler (2000)

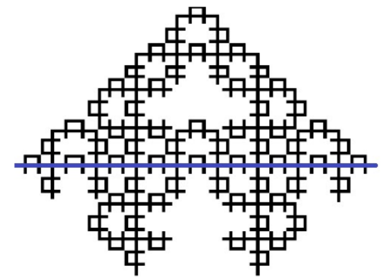
$$\frac{\partial^{\gamma} p_{D_f}}{\partial t_D^{\gamma}} = \frac{1}{r_D^{d_{mf}-1}} \frac{\partial}{\partial r_D}\left(r_D^{\beta} \frac{\partial p_{D f}}{\partial r_D} \right)$$

Concepto de flujo fractal continuo

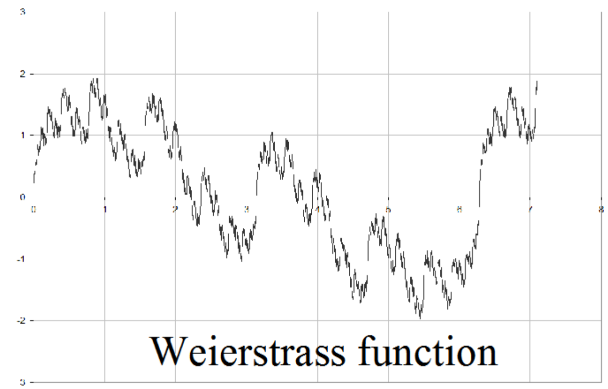


Fractales de misma dimensión fractal pero de diferente conectividad

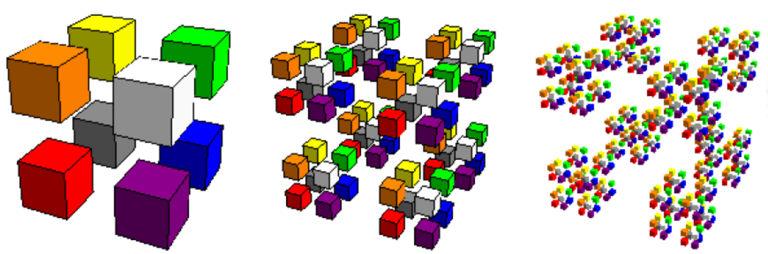




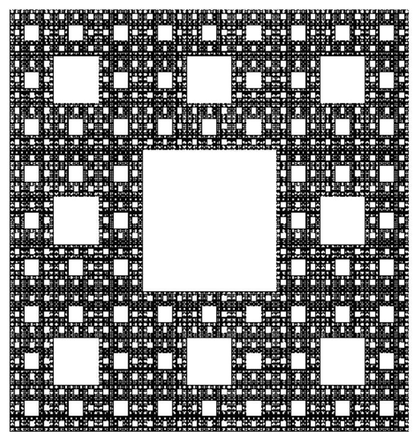
$$D_M = \frac{\ln 8}{\ln 3} = 1.893$$



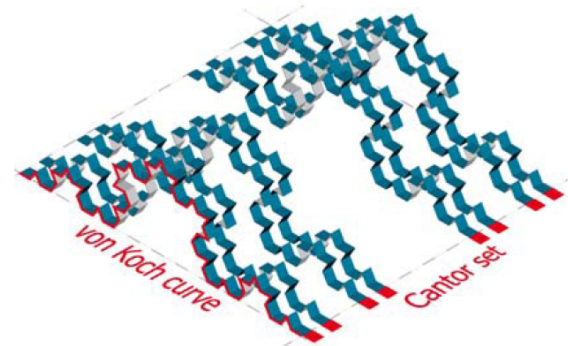
$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x),$$



$$D_M = \frac{\ln 8}{\ln 3} = 1.893$$

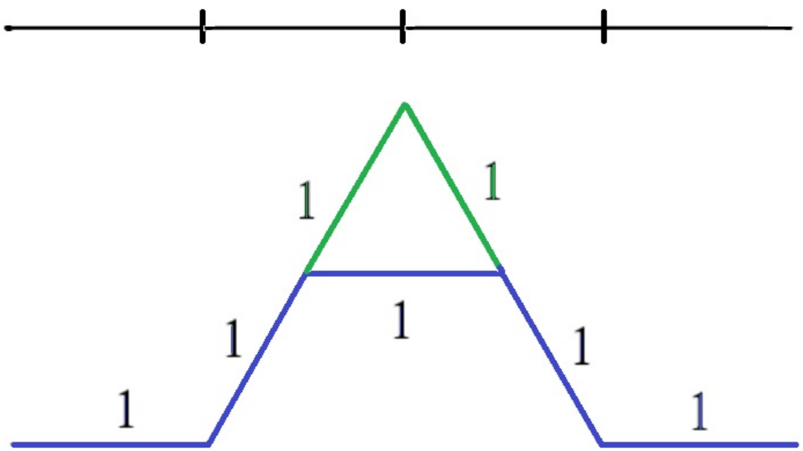


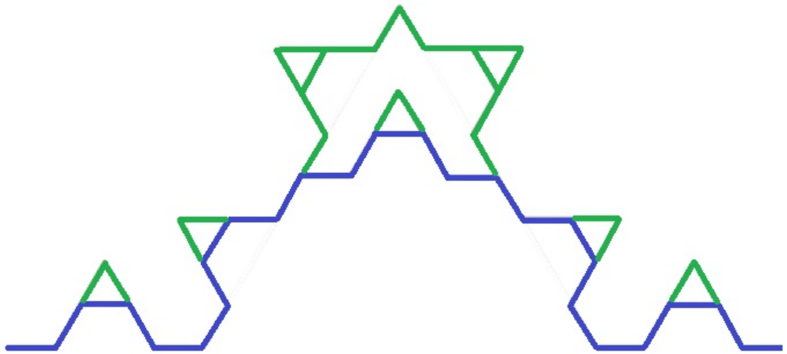
$$D_M = \frac{\ln 8}{\ln 3} = 1.893$$



$$D_M = \frac{\ln 8}{\ln 3} = 1.893$$

BASIC FRACTAL DIMENSIONS





- Similarity dimension

$$N = L^{D_M}$$ $$D_M = \frac{\ln 7}{\ln 4} = 1.404...$$

- Chemical dimension

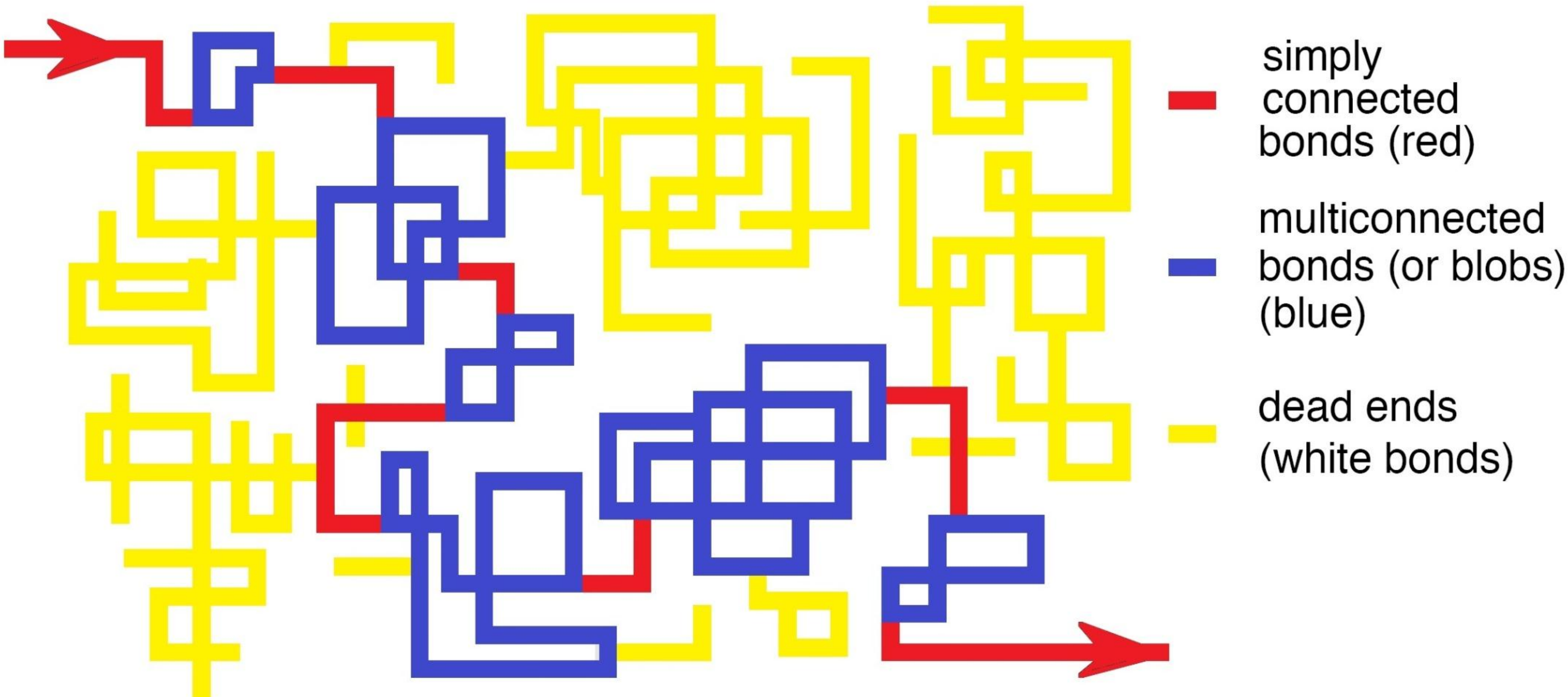
$$N = \ell^{d_\ell}$$ $$d_\ell = \frac{\ln 7}{\ln 5} = 1.209...$$

ℓ - shortest path

- Dimension of the shortest path

$$\ell = L^{d_{\min}}$$ $$d_{\min} = \frac{\ln 5}{\ln 4} = 1.1609..$$

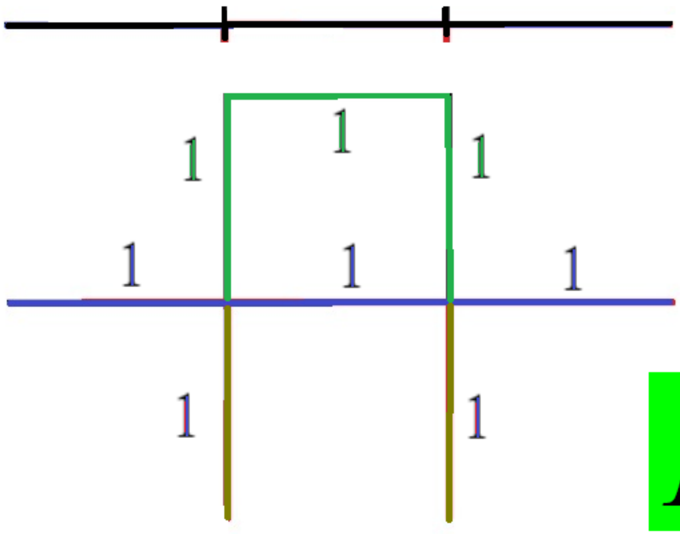
BASIC FRACTAL DIMENSIONS



$$N_{red} \propto L^{D_{red}} \qquad N_{red} + N_{blue} \propto L^{D_B}$$

$$N_{white} \propto L^{D}$$

BASIC FRACTAL DIMENSIONS



- Similarity dimension

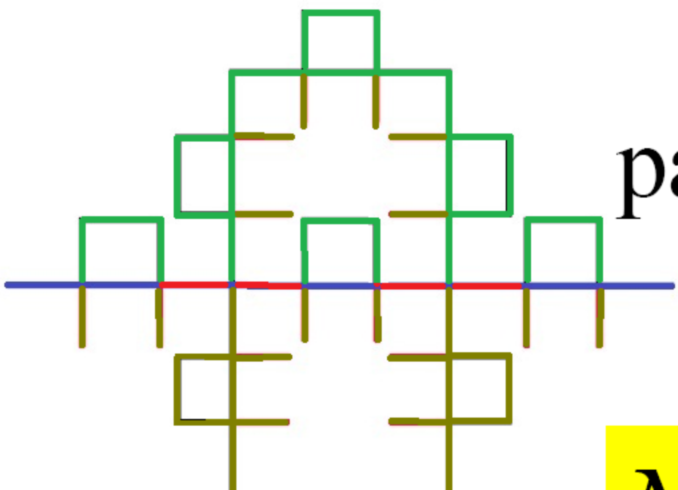
$$N = L^{D_M} \qquad D_M = \frac{\ln 8}{\ln 3} = 1.893\ldots$$

- Chemical dimension

$$N = \ell^{d_\ell} \qquad d_\ell = D_M = \frac{\ln 8}{\ln 3} = 1.893\ldots$$

- Dimension of the shortest path

$$\ell = L^{d_{\min}} \qquad d_{\min} = 1$$



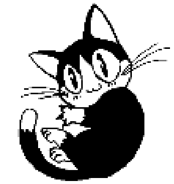
- Backbone dimension

$$N_{bb} = L^{D_{bb}} \qquad D_{bb} = \frac{\ln 6}{\ln 3} = 1.6309\ldots$$

Red bonds dimension $D_{red} = \frac{\ln 2}{\ln 3} = 0.631$

❖ The dead ends are the parts that do not carry any current

SPECTRAL DIMENSION



2.3. *Fracton (spectral) dimensionality and the Alexander–Orbach conjecture*

The problem of diffusion is closely related to the density of states [91] in the substrate. This can be best understood from the relation [4, 92] between the density of states $\mathcal{N}(\epsilon)$ and the probability $P(0, N)$ of the random walker returning to the origin

$$P(0, N) = \int_0^\infty \mathcal{N}(\epsilon) \exp(-\epsilon N)\, \mathrm{d}\epsilon. \tag{2.8 a}$$

After performing N steps, the number of sites a random walker has visited is proportional to the volume $R^{d_f} \sim N^{d_f/d_w}$. Therefore the probability of returning to the origin scales as [45]

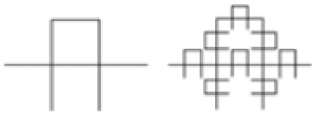
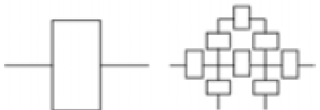
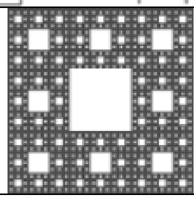
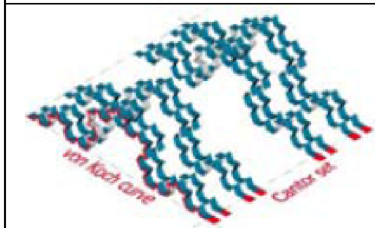
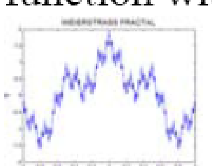
$$P(0, N) \sim 1/R^{d_f} \sim N^{-\mathrm{d_f}/d_w}. \tag{2.8 b}$$

Using this result in (2.8 *a*), one obtains the following expression for the density of states on the substrate:

$$\mathcal{N}(\epsilon) \sim \epsilon^{d_f/d_w - 1} \equiv \epsilon^{d_s/2-1}. \tag{2.9}$$

This is similar to the usual expression for Euclidean space, $\mathcal{N}(\epsilon) \sim \epsilon^{d/2-1}$, except that d is replaced by $2d_f/d_w$. The ratio $2d_f/d_w$ is then identified as the relevant dimension for the density of states in a substrate, and is called *fracton* dimensionality

$$D_M = \frac{\ln 8}{\ln 3} = 1.893$$

Fractal	d_ℓ	$d_{\min}$	D_{bb}	D_{red}	d_s
	D_M	1	$\frac{\ln 6}{\ln 3} = 1.631$	$\frac{\ln 2}{\ln 3} = 0.631$	?
	$\frac{\ln 8}{\ln 5} = 1.292$	$\frac{\ln 5}{\ln 3} = 1.465$	D_M	$\frac{\ln 2}{\ln 3} = 0.631$	?
	D_M	1	D_M	-	1.805
	3	$\frac{\ln 2}{\ln 3} = 0.631$	-	-	2?
	2? 1?	$\frac{\ln 2}{\ln 3} = 0.631$ $\frac{\ln 4}{\ln 3} = 1.262$	-	$\frac{\ln 2}{\ln 3} = 0.631$?	?
Graph of Weierstrass function with 	1	D_M	D_M	-	$2H$?

The Concept of Fractal Continua

A. S. Balankin and B. Espinoza Elizarraraz (2012)

Topological and Chemical Dimensions **How points within an object are connected**
Fractal Dimension **How many new pieces we see when we look at a fine resolution**
Spectral Dimension **How many degrees of freedom has each point**

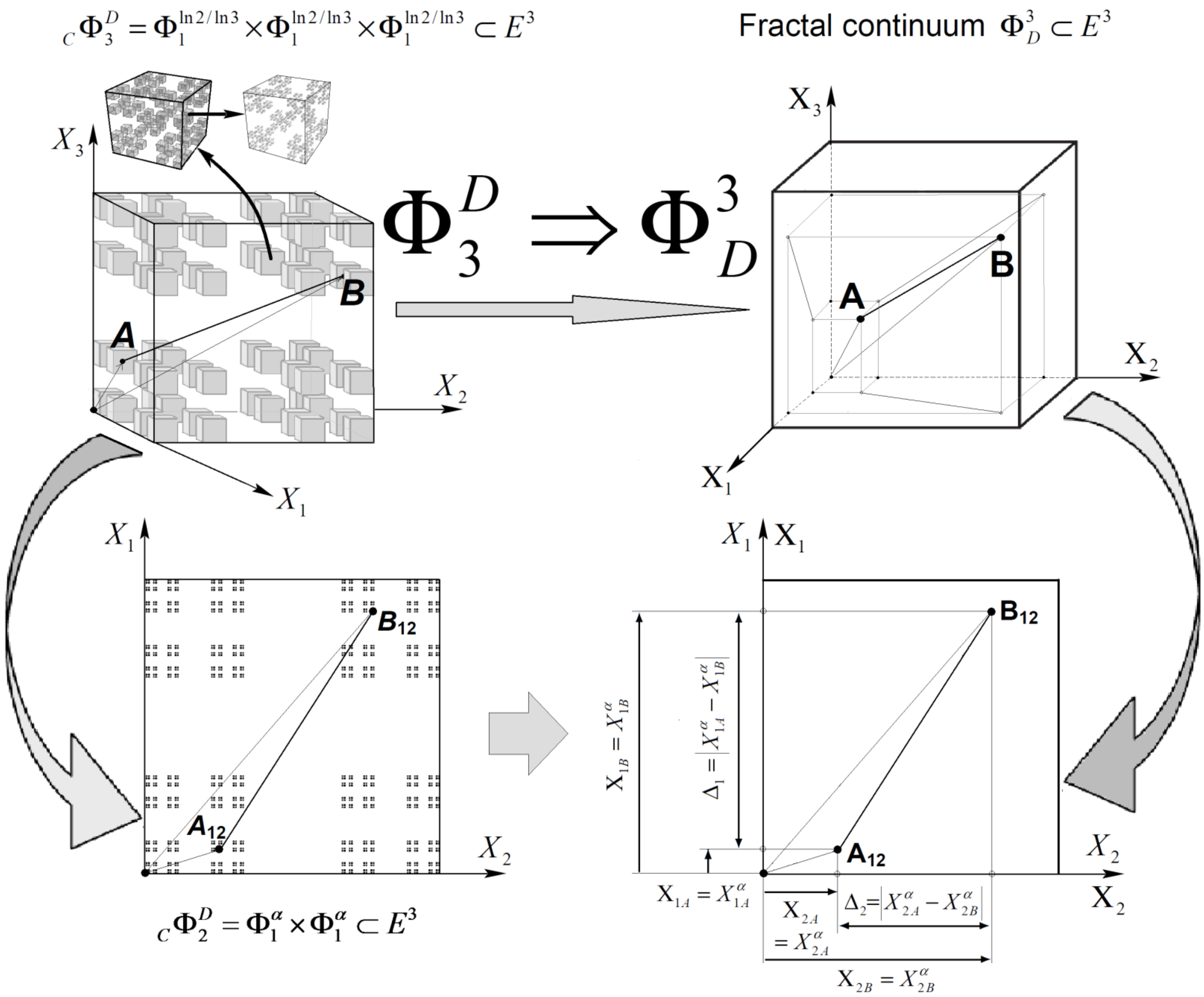
Strictly speaking, a fractal with $D<3$ cannot continuously fill an embedding Euclidean space E^3. Nevertheless, the **fractal continuum** Φ_D^3 can be defined as a **three-dimensional region of Euclidean space E^3 filled with continuous matter** (leaving no pores or empty spaces) such that its properties (*e.g.* density $\rho(x_i)$, displacements $v_j(x_i)$, *etc.*) are describable by the continuous (or at least piecewise continuous) and differentiable functions of the space and time variables, while the extensive properties of any cubic (or spherical) region $W \in \Phi_D^3 \subset E^3$ of the size L obey the fractal scaling law

$$Y = \int_W y(x_i) dV_D = \int_W y(x_i) c_3(x_i, D) dV_3 = y_0 (L/\ell_0 + 1)^D,$$

where Y is any extensive property, e.g. mass $Y = M(L)$, while $y(x_i)$ is the corresponding mass density distribution.

Concepto de flujo fractal continuo

Metrica



Concepto de flujo fractal continuo

Derivativa

Taking into account that fractal continuum Φ_D^3 continuously fill the embedding Euclidean space E^3, from Eqs. (2)-(4) follows that the distance between two points $A, B \in \Phi_D^3 \subset E^3$ can be defined as $\Delta(A,B) = \sqrt{\sum_i^3 \Delta_i^2(a_i, b_i)}$, where

$$\Delta_i(a_i, b_i) = \left| \int_{a_i}^{b_i} d^{\zeta_i} x_i \right| = \ell_0 \left| \left(1 + \frac{a_i}{\ell_0}\right)^{\zeta_i} - \left(1 + \frac{b_i}{\ell_0}\right)^{\zeta_i} \right|,$$

where the co-dimensions ζ_i are defined by Eq. (4). It is a straightforward matter to see that $\Delta(A,B)$ satisfies all conventional criteria required of **metrics**: a) $\Delta(A,B) \geq 0$, b) $\Delta(A,B) = \Delta(B,A)$, c) $\Delta(A,A) = 0$, d) if $\Delta(A,B) = 0$ than $A = B$, e) the triangle inequality $\Delta(A,B) + \Delta(A,C) \geq \Delta(B,C)$. This permits to define the partial **local fractional derivatives** in Φ_D^3 as

$$\nabla_i^H f = \lim_{x_i \to x'_i} \frac{f(x_i') - f(x_i)}{\Delta(x_i', x_i)} = \left|1 + \frac{x_i}{\ell_0}\right|^{1-\zeta_i} \frac{\partial}{\partial x_i} f(x) = \chi^{(i)} \nabla_i f,$$

where $\nabla_i = \partial / \partial x_i$ denotes the conventional partial derivative. Notice that the Hausdorff derivative has the dimension of conventional derivative, that is $[\nabla_i^H] = [L^{-1}]$. Accordingly, **the Hausdorff del operator** is defined as

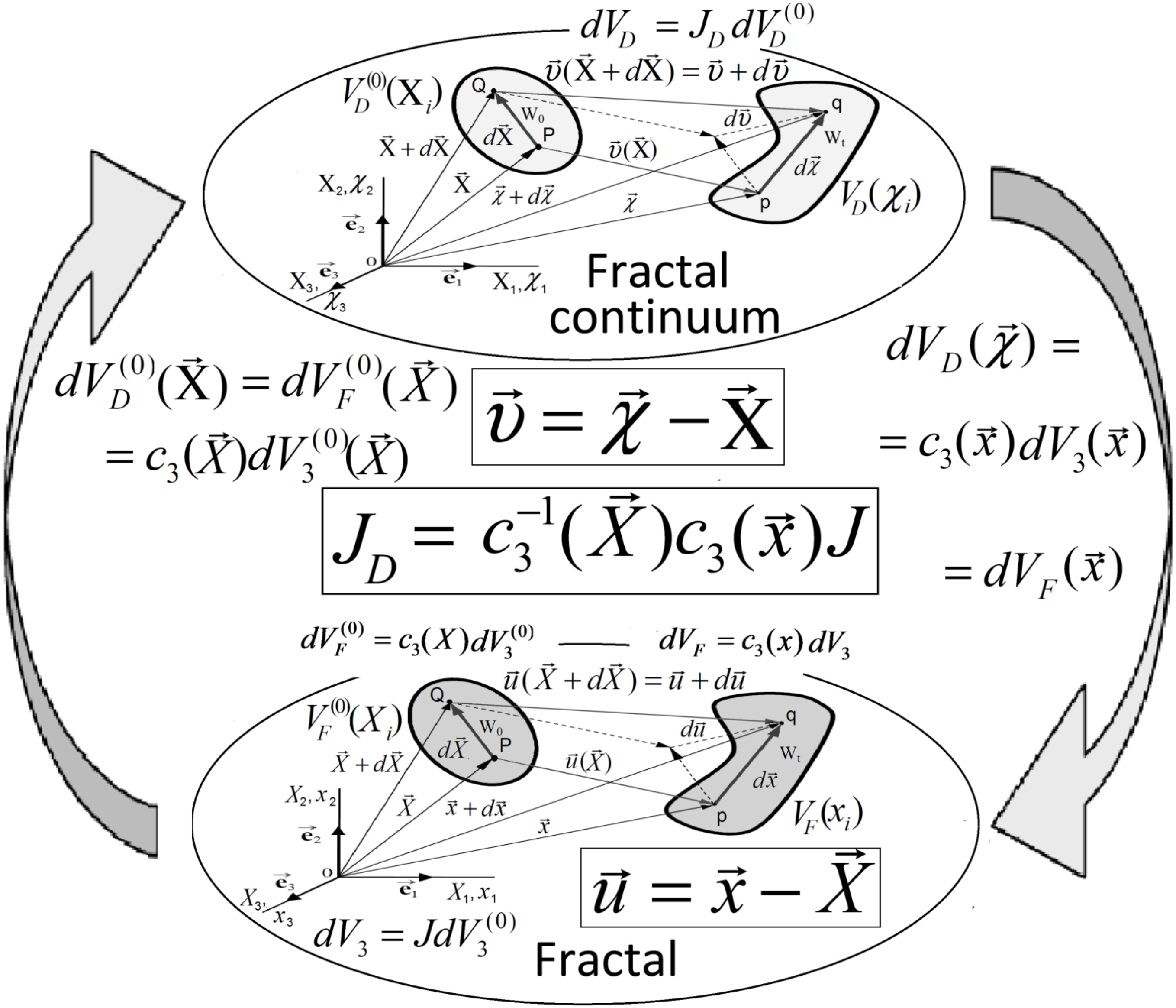
$$\vec{\nabla}^H = \vec{e}_1 \nabla_1^H + \vec{e}_2 \nabla_2^H + \vec{e}_3 \nabla_3^H,$$

where $\vec{e}_i$ denote the Cartesian directions.

$$\chi^{(i)} = \ell_i^{\zeta_i - 1} / c_1^{(i)}(x_i) = (x_i/\ell_i + 1)^{1-\zeta_i},$$

Concepto de flujo fractal continuo

Cinemateca



Hidrodinámica de flujo fractal continuo

PHYSICAL REVIEW E **85**, 025302(R) (2012)

Hydrodynamics of fractal continuum flow

Alexander S. Balankin and Benjamin Espinoza Elizarraraz
Grupo "Mecánica Fractal," Instituto Politécnico Nacional, México Distrito Federal, México 07738
(Received 7 December 2011; revised manuscript received 25 January 2012; published 13 February 2012)

A model of fractal continuum flow employing local fractional differential operators is suggested. The generalizations of the Green-Gauss divergence and Reynolds transport theorems for a fractal continuum are suggested. The fundamental conservation laws and hydrodynamic equations for an anisotropic fractal continuum flow are derived. Some physical implications of the long-range correlations in the fractal continuum flow are briefly discussed. It is noteworthy to point out that the fractal (quasi)metric defined in this paper implies that the flow of an isotropic fractal continuum obeying the Mandelbrot rule of thumb for intersection is governed by conventional hydrodynamic equations.

Ecuación de Navier-Stokes para flujo fractal continuo

$$\rho_c\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla^H)\vec{u}\right] = -\mathrm{grad}_H\, p + \mu\Delta_H\vec{u} + \left(\lambda + \frac{\mu}{3}\right)\mathrm{grad}_H \mathrm{div}_H\vec{u} + \vec{f}$$

Hidrodinámica de flujo fractal continuo

PHYSICAL REVIEW E **85**, 056314 (2012)

Map of fluid flow in fractal porous medium into fractal continuum flow

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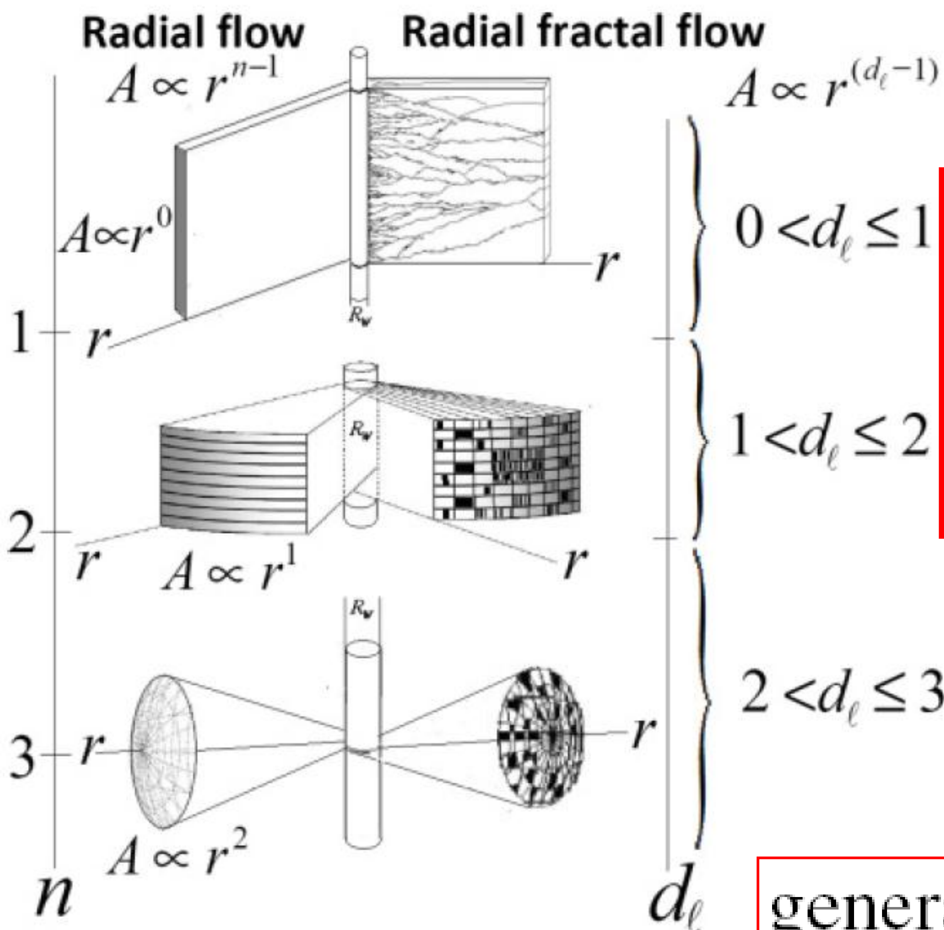
(Received 7 March 2012; published 30 May 2012)

This paper is devoted to fractal continuum hydrodynamics and its application to model fluid flows in fractally permeable reservoirs. Hydrodynamics of fractal continuum flow is developed on the basis of a self-consistent model of fractal continuum employing vector local fractional differential operators allied with the Hausdorff derivative. The generalized forms of Green-Gauss and Kelvin-Stokes theorems for fractional calculus are proved. The Hausdorff material derivative is defined and the form of Reynolds transport theorem for fractal continuum flow is obtained. The fundamental conservation laws for a fractal continuum flow are established. The Stokes law and the analog of Darcy's law for fractal continuum flow are suggested. The pressure-transient equation accounting the fractal metric of fractal continuum flow is derived. The generalization of the pressure-transient equation accounting the fractal topology of fractal continuum flow is proposed. The mapping of fluid flow in a fractally permeable medium into a fractal continuum flow is discussed. It is stated that the spectral dimension of the fractal continuum flow d_s is equal to its mass fractal dimension D, even when the spectral dimension of the fractally porous or fissured medium is less than D. A comparison of the fractal continuum flow approach with other models of fluid flow in fractally permeable media and the experimental field data for reservoir tests are provided.

Fractal Continuum Flow

A. S. Balankin and B. A. Espinoza Elizarraraz (2012)



$$c\mu_c \frac{\partial p}{\partial t} = \mathrm{div}_H \left[K_{ii}^{(c)} \mathrm{grad}_H (p - h_g) \right]$$
$$= \sum_i^3 K_{ii}^{(c)} \left(\frac{x_i}{\ell_i} + 1 \right)^{2(1+d_i-D)} \left[\left(\frac{\partial^2 (p - h_g)}{\partial x_i^2} \right) + \frac{1 + d_i - D}{x_i + \ell_i} \left(\frac{\partial (p - h_g)}{\partial x_i} \right) \right], \quad (92)$$

generalized fractional Laplacian

$$\Delta_D^H \psi = \sum_i^3 (\chi^{(i)})^2 \left[\left(\frac{\partial^2 \psi}{\partial x_i^2} \right) + \frac{\alpha_i - D + d_i}{x_i + \ell_i} \left(\frac{\partial \psi}{\partial x_i} \right) \right]$$

Hidrodinámica de flujo fractal continuo

TABLE III. Relations between the properties of fractal continuum flow, the properties of the fractally permeable medium, fluid, and the flow dynamics and geometry.

Parameter	Flow in porous and/or fissured medium: Permeable medium	Fluid	Fluid flow	Fractal continuum flow
Porosity	ϕ			
Characteristic size ℓ_0	Eq. (2)			Eqs. (1) and (12)
Tensor of absolute permeability	K_{ij}			
Relative permeability		k_{ij}		
Effective property of continuum flow related to the medium permeability				$K_{ij}^{(c)} = f(K_{ij}, k_{ij})$
Fractal properties	See Table I		Determined by the fractal geometry of permeable medium	See Table II
Flow dimension			See Fig. 3	d_ℓ
Density	$\rho_m = \rho_0(1-\phi)$	ρ_f	$\rho_f(p, x_i, t)$	Eq. (74)
Compressibility	c_m	c_f	Eq. (85)	Eqs. (90) and (91)
Constitutive law		Newtonian	Eq. (71)	Eq. (72)
Kinematic viscosity			$\eta_f = \eta$	$\eta_c = \eta$
Dynamics viscosity			$\mu_f = \rho_f \eta$	Eq. (75)
Coefficient of internal viscosity			λ_f	$\lambda_c = \phi \lambda_f$
Flow velocity (u)			Eq. (45)	Eqs. (45) and (52)
Darcian velocity			Eq. (77)	Eq. (80)
Hydrostatic head			$gz(x, y)$	Eq. (62)

Hidrodinámica de flujo fractal continuo

TABLE II. Fractal properties of porous medium and the corresponding fractal continua.

Fractal porous medium (Φ^D)		Fractal continuum			
Property	Scaling exponent	Characteristic	Continuum flow ${}^{D}_{3}\Phi^3_D \subset E^3$	Continuum flow ${}^{D}_{d_\ell}\Phi^3_D \subset E^3$	Diffusion ${}^{d_s}_{d_\ell}\Phi^3_D \subset E^3$
Mass scaling	D	Fractal metric	Fractal mass dimension associated with the box-counting quasimeasure Eqs. (8)–(10)		
Connectivity	d_ℓ	Topology	$d_\ell = 3$	Fractional topological dimension of fractal continuum [see Eqs. (23) and (24)]	
Shortest paths scaling	$d_{\min} = D/d_\ell$	Tortuosity		Accounted in constitutive laws	
Intersection with plane	Fractal dimensions of intersections d_i $i = 1,2,3$	Density of states on the intersection with Cartesian plane (x_j, x_k)	Fractal dimension of fractal continuum intersection with Cartesian plane (x_j, x_k)		
Intersection with line	Fractal dimension of intersection		Not defined		
$D - d_i$	$\zeta_i = D - d_i$	Density of states in the direction of the normal to intersection with Cartesian plane (x_j, x_k)		Order of the Hausdorff derivative (19)	
Roughness of pore and/ or fracture surfaces	D_S	Permeability (resistance)		Accounted in constitutive laws	
Spectral dimension	$d_s = 2D/D_W = \frac{2D}{2+\theta}$ [a]	Dimension of Lagrangian [14]	$d_s = D$ [b]		Order of the Hausdorff time derivative [see Eqs. (35) and (36)]

[a] Alexander-Orbach law [41] associated with the fractal Einstein law related the electrical resistivity exponent to the fractal dimensions of medium and random walk $\vartheta = D_W - D$ [42].
[b] In the case of Darcian fractal continuum flow the Alexander-Orbach law fails, while $d_s = D$, such that the fractal Einstein law obeys the generalized form $\vartheta = 0.5D_W(2 - D)$ (see Ref. [44]).

Flujo fractal continuo transitoria

$$c\mu_c \frac{\partial p}{\partial t} = \mathrm{div}_H\left[K_{ii}^{(c)} \mathrm{grad}_H(p - h_g)\right]$$
$$= \sum_{i}^{3} K_{ii}^{(c)} \left(\frac{x_i}{\ell_i} + 1\right)^{2(1+d_i-D)} \left[\left(\frac{\partial^2 (p - h_g)}{\partial x_i^2}\right) + \frac{1 + d_i - D}{x_i + \ell_i}\left(\frac{\partial (p - h_g)}{\partial x_i}\right)\right], \quad (92)$$

$$\frac{\partial p_D}{\partial t_D} = \frac{\chi_D^{(r)}}{(r_D + \ell_D)^{n-1}} \frac{\partial}{\partial r_D}\left[(r_D + \ell_D)^{n-1} \chi_D^{(r)} \frac{\partial}{\partial r_D} p_D\right]$$
$$= \left(\frac{r_D}{\ell_D} + 1\right)^{d_r+2-D-n} \frac{\partial}{\partial r_D}\left[\left(\frac{r_D}{\ell_D} + 1\right)^{d_r-D+n} \frac{\partial}{\partial r_D} p_D\right]$$
$$= \left(\frac{r_D}{\ell_D} + 1\right)^{2(d_r+1-D)} \left(\frac{\partial^2 p_D}{\partial r_D^2} + \frac{d_r + n - D}{r_D + \ell_D} \frac{\partial p_D}{\partial r_D}\right), \quad (94)$$

$$c\mu_c \frac{\partial p}{\partial t} = \sum_{i}^{3} K_{ii}^{(c)} \left(\frac{x_i}{\ell_i} + 1\right)^{2(1+d_i-D)} \left[\left(\frac{\partial^2 (p - h_g)}{\partial x_i^2}\right) + \frac{\alpha_i + d_i - D}{x_i + \ell_i}\left(\frac{\partial (p - h_g)}{\partial x_i}\right)\right], \quad (96)$$

TABLE IV. Dimensionless variables for the pressure-transient equations at a constant production rate. Here r_D is the dimensionless radial distance from the pumping well, t_D is the dimensionless time, p_D is the dimensionless pressure drop, p_0 is the initial pressure, R_W is the well radius, Q is the volumetric flow rate, and the coefficients κ_n are defined as follows: $\kappa_1 = H R_W, \kappa_2 = 2\pi H, \kappa_3 = 4\pi$, while $\kappa_n^D = 2^{D^*-1}\pi R_W^{3-D^*}$ and $\kappa_n^\theta = R_W^{3-D_n+\theta}$.

Model and pressure-transient equation	Dimensionless variables		
	$t_D = t/\tau_c$	$p_D = (p_0 - p)/p_c$	$r_D = r/r_c$
Euclidean, Eq. (86)	$\tau_c = \frac{c\mu\phi R_W^2}{K_r}$	$p_c = \frac{Q\mu}{\kappa_n R_W^{n-2} K_r}$	$r_c = R_W$
Barker, Eq. (87)	$\tau_c = \frac{c\mu\phi R_W^2}{K_r}$	$p_c = \frac{Q\mu}{\kappa_n^D R_W^{D^*-2} K_r}$	$r_c = R_W$
Chang and Yortsos, Eq. (89)	$\tau_c = \frac{c\mu\phi R_W^{2+\theta}}{K_r}$	$p_c = \frac{Q\mu}{\kappa_n^\theta R_W^{D_n-\theta-2} K_r}$	$r_c = R_W$
${}_{3}^{D}\Phi_D^3$, Eq. (94)	$\tau_c = \frac{c\mu\phi r_c^2}{K_r}$	$p_c = \frac{Q\mu}{\kappa_n r_c^{n-2} K_r}$	$r_c = \frac{R_W}{2\zeta-1}\left(\frac{R_W}{\ell_0} + 1\right)^{\zeta-1}$
${}_{d_\ell}^{D}\Phi_D^3$, Eq. (97)	$\tau_c = \frac{c\mu\phi r_c^2}{K_r}$	$p_c = \frac{Q\mu}{4\pi r_c K_r}$	$r_c = \frac{R_W}{2\zeta-1}\left(\frac{R_W}{\ell_0} + 1\right)^{\zeta-1}$

MODELOS DE PRESION EN MEDIO FRACTAL ANISOTROPICO

A. S. Balankin and B. Espinoza, Phys. Rev. E 85, 056314 (2012).

Ecuación de presión no lineal 3D

$$\phi\rho_f c_t \frac{\partial p}{\partial t} = \nabla^H \cdot \left[\frac{\rho_f \mathbf{K}^{(c)}}{\mu_f} \nabla^H (p - h_g) \right]$$

$$c_t = c_f + \frac{\phi^0}{\phi} c_R$$

$$\phi = \phi^0 e^{c_R (p - p^0)}$$

$$\rho = \rho^0 e^{c_f (p - p^0)}$$

Donde la cabeza gravitacional esta dada como

$$h_g = h_0 - (D - d_z)\rho_f g \ell_z \left(\frac{z}{\ell_z} + 1 \right)^{D - d_z}$$

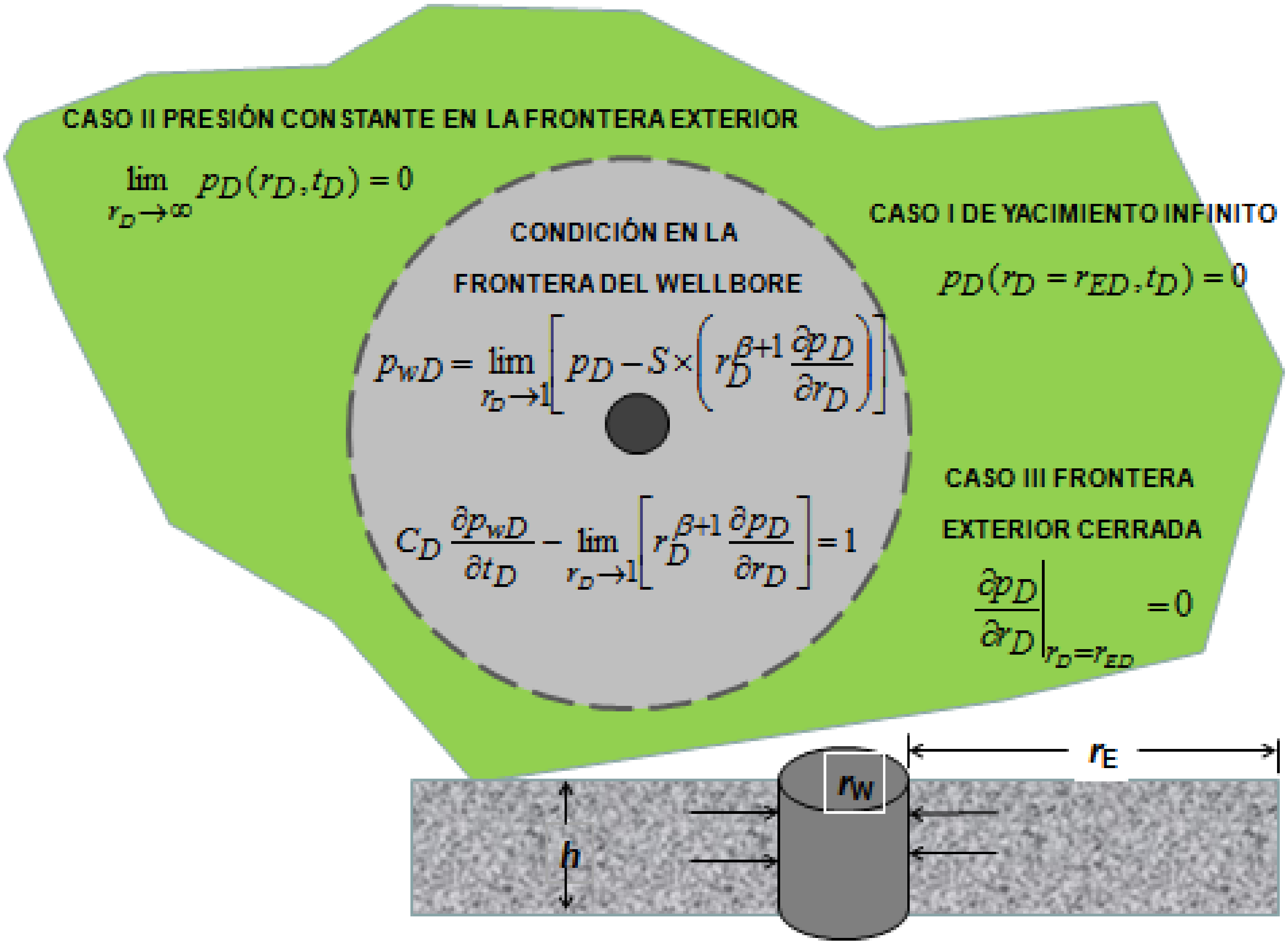
Propiedades del medio	
Símbolo	Descripción
ϕ	Porosidad
c_R	Compresibilidad isotérmica del la roca
K_{ij}	Tensor de permeabilidad

Símbolo	Descripción
h_0	Cabeza gravitacional de referencia
g	Magnitud de la aceleración de la gravedad
$\nabla_k^H = \left(\frac{x_k}{\ell_k} + 1 \right)^{1 - \zeta_k} \frac{\partial}{\partial x_k}$	Derivada fraccional parcial local

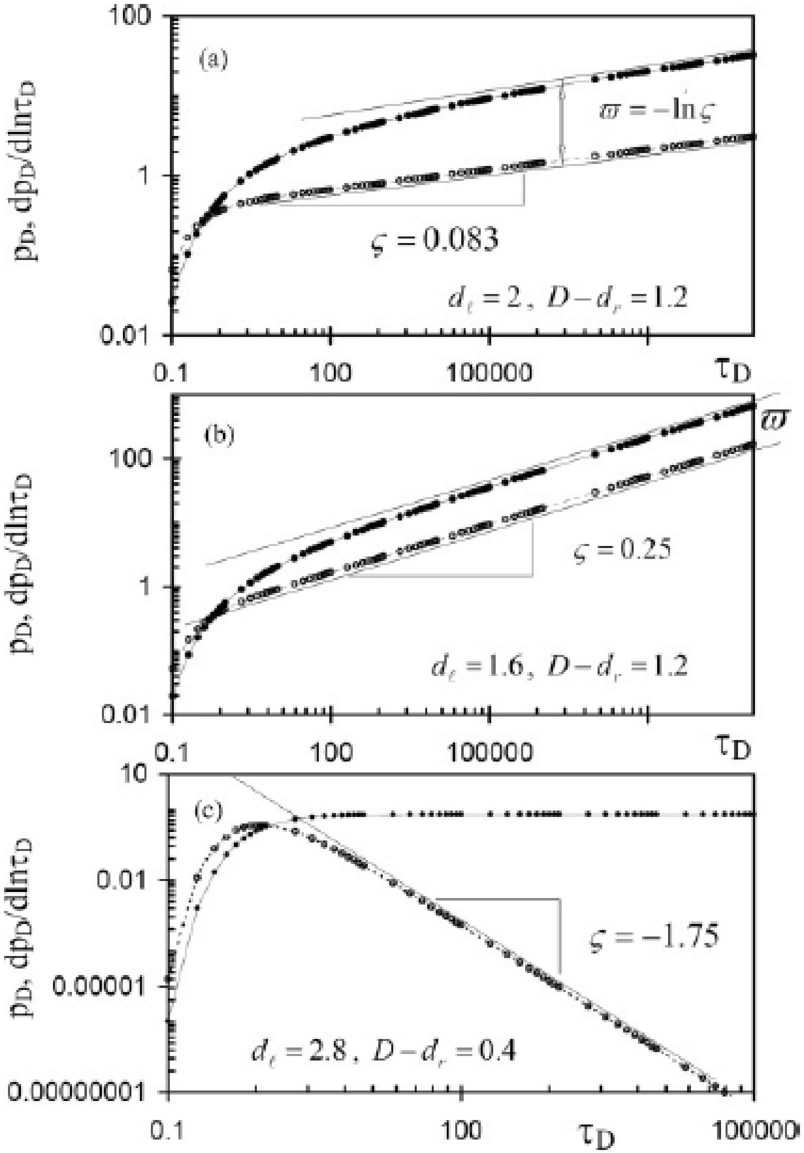
Propiedades del fluido	
Símbolo	Descripción
$\rho(\mathbf{r}, t)$	Densidad del fluido
c_f	Compresibilidad isotérmica del fluido
μ	Viscosidad del fluido

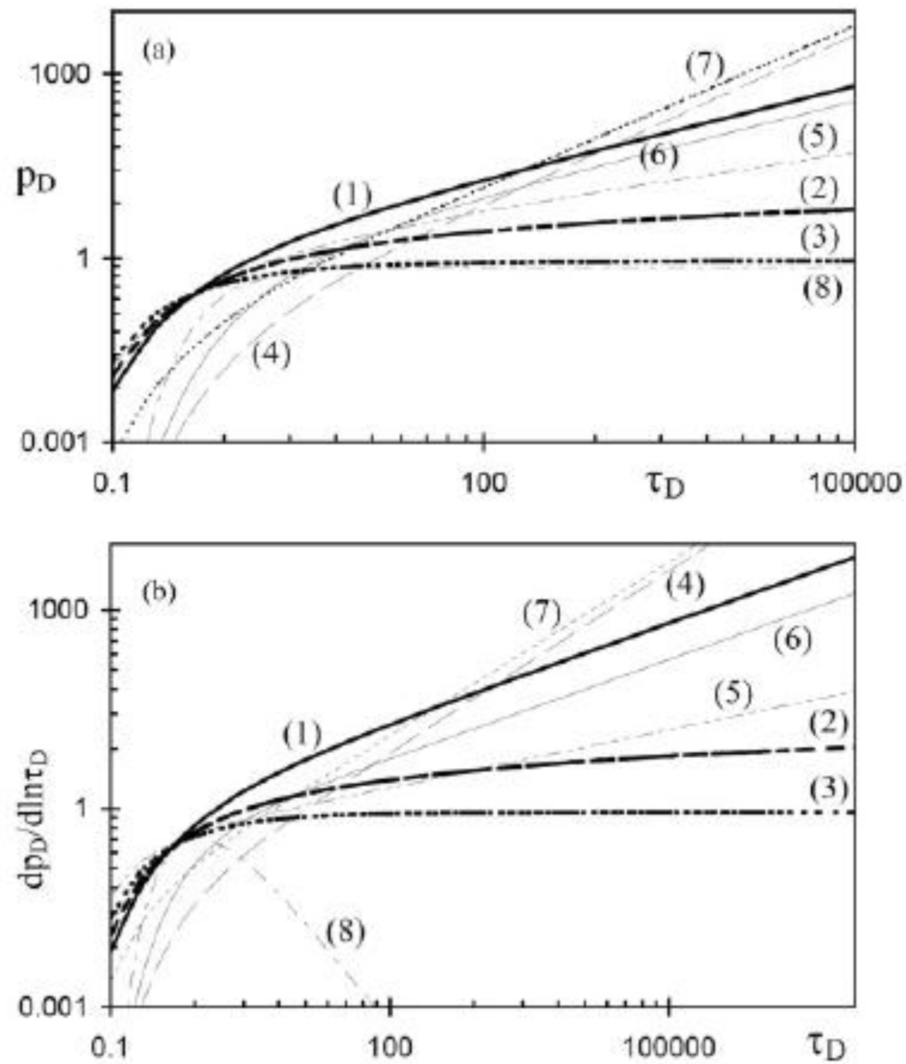
$$\zeta_k = D - d_k$$

Hidrodinámica de flujo fractal continuo



Flujo transitorio – CURVAS TIPO





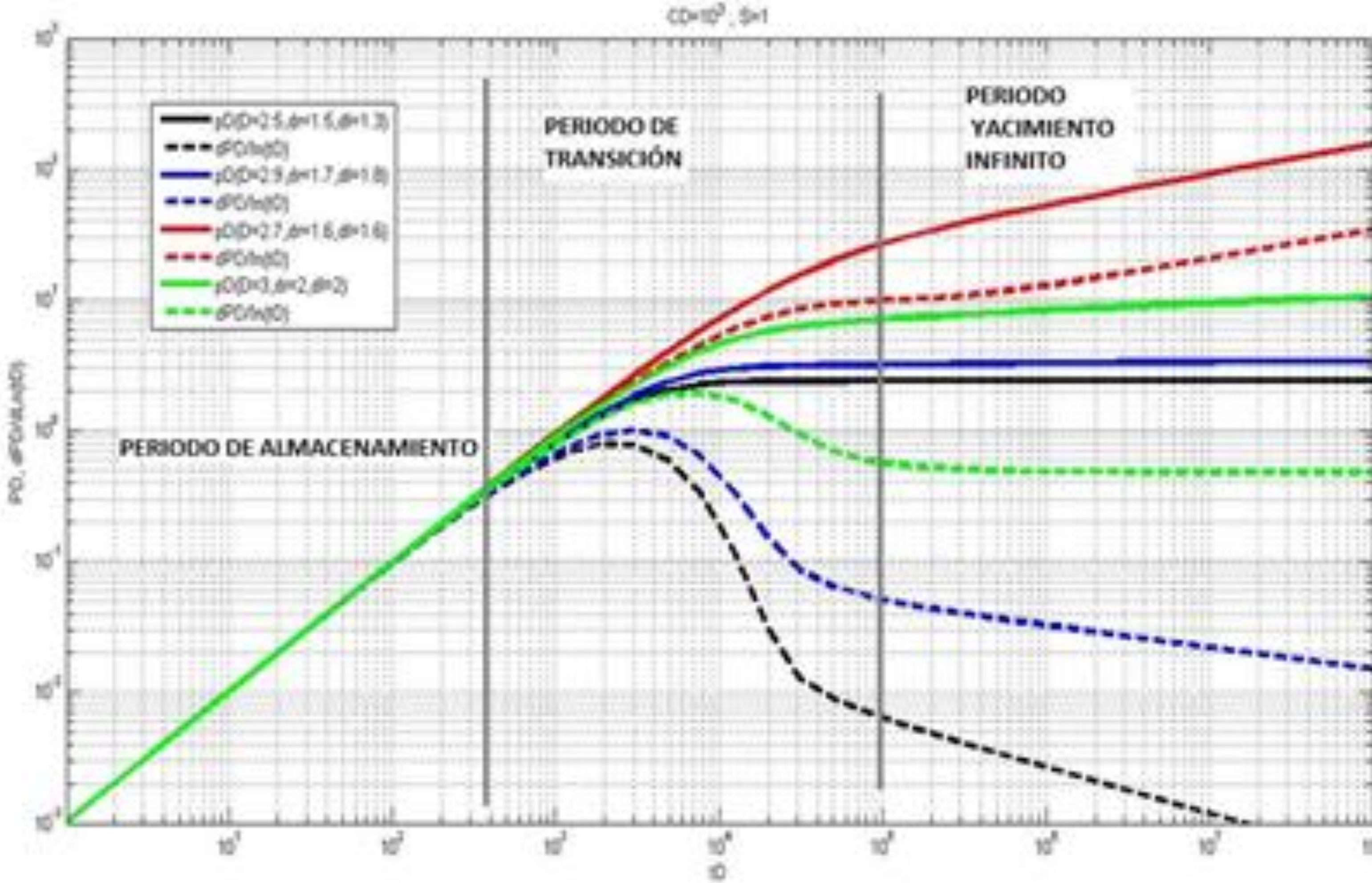
2. MODELO PARA UN YACIMIENTO FRACTAL INFINITO CON DAÑO Y ALMACENAMIENTO

I) Yacimiento infinito	- El pozo esta inmerso en un yacimiento de extensión infinita. - A puntos muy lejanos del pozo, la presión del yacimiento permanece constante
II) Yacimiento de frontera cerrada	- El pozo esta localizado en el centro del yacimiento y no hay flujo a través de la frontera exterior.
III) Presión constante en la frontera cerrada exterior	- El pozo se localiza en el centro del yacimiento y la presión permanece constante a lo largo de la frontera exterior.

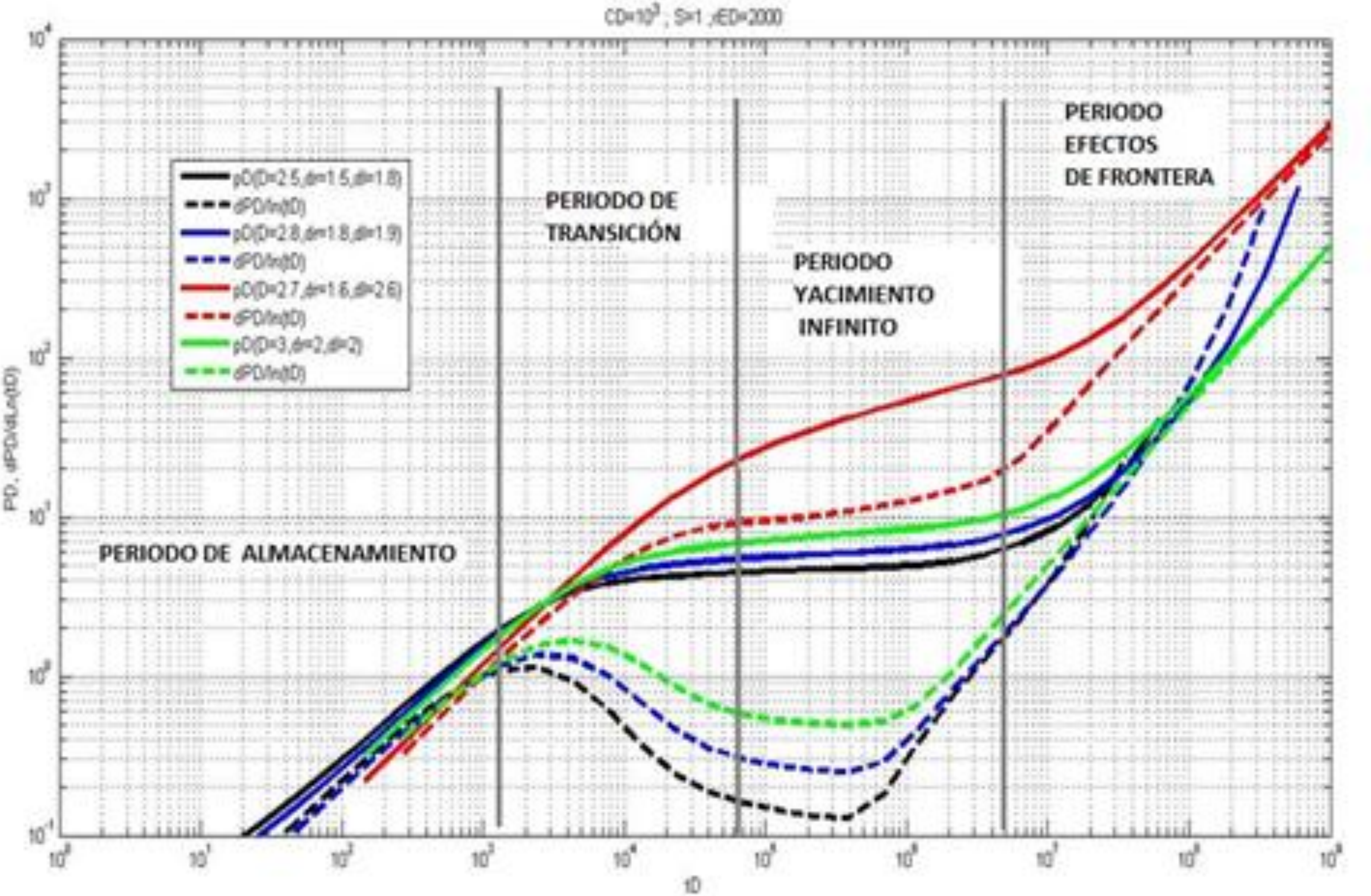
Para estos tres casos:	- La presión inicial del yacimiento permanece constante en cualquier punto de la formación a un tiempo inicial. - La tasa de flujo en la superficie del pozo es constante. - Los efectos de daño solo sedan en la frontera del pozo (wellbore)

Modelo a tasa de producción constante con daño y almacenamiento

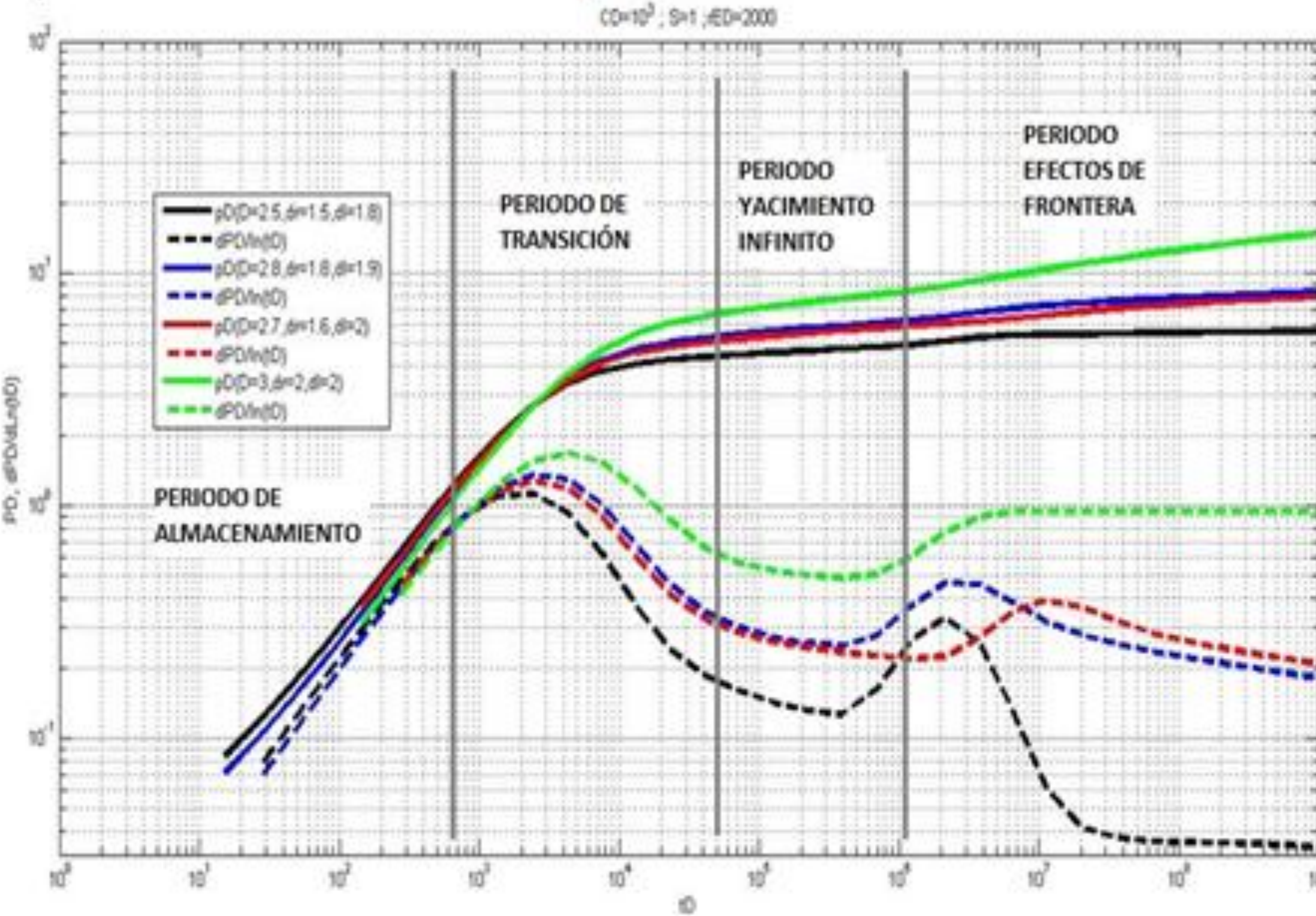
$$\bar{p}_{wD}(s)=\frac{BesselK\left(\frac{\beta}{\gamma},\frac{2\sqrt{s}}{\gamma}\right)+S\times\sqrt{s}BesselK\left(\frac{\beta}{\gamma}-1,\frac{2\sqrt{s}}{\gamma}\right)}{C_D s^2\left[BesselK\left(\frac{\beta}{\gamma},\frac{2\sqrt{s}}{\gamma}\right)+S\times\sqrt{s}BesselK\left(\frac{\beta}{\gamma}-1,\frac{2\sqrt{s}}{\gamma}\right)\right]+s^{3/2}BesselK\left(\frac{\beta}{\gamma}-1,\frac{2\sqrt{s}}{\gamma}\right)}$$



I) Circulo acotado / cerrado

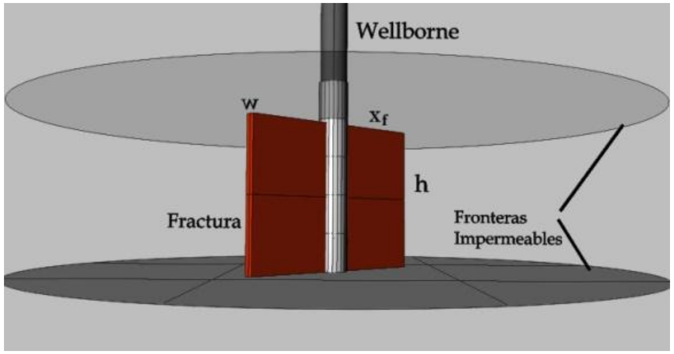


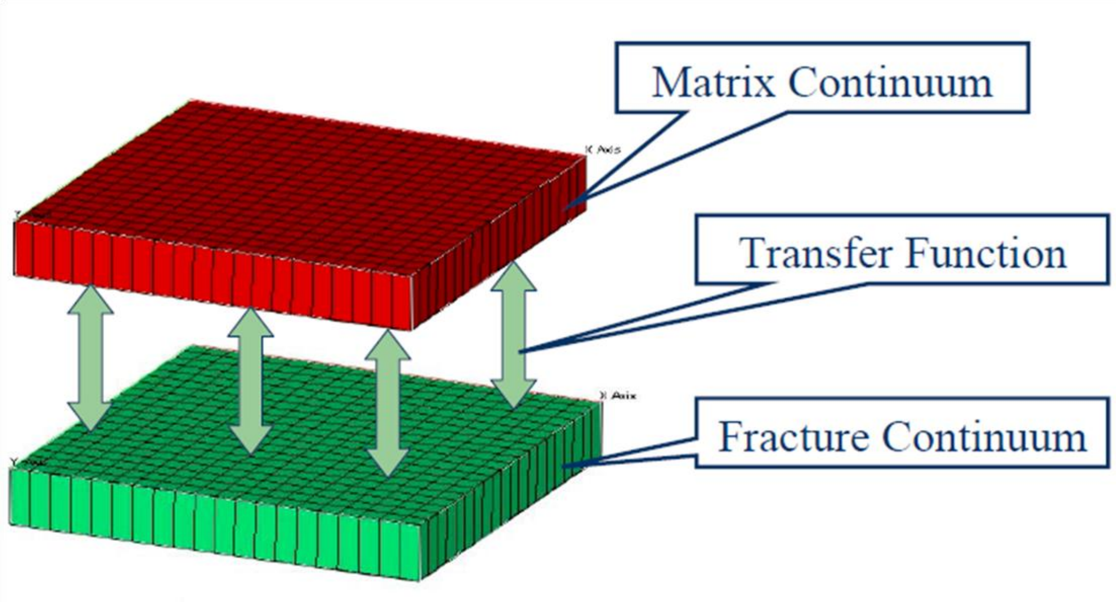
II) Circulo acotado / cerrado a presión constante en la frontera

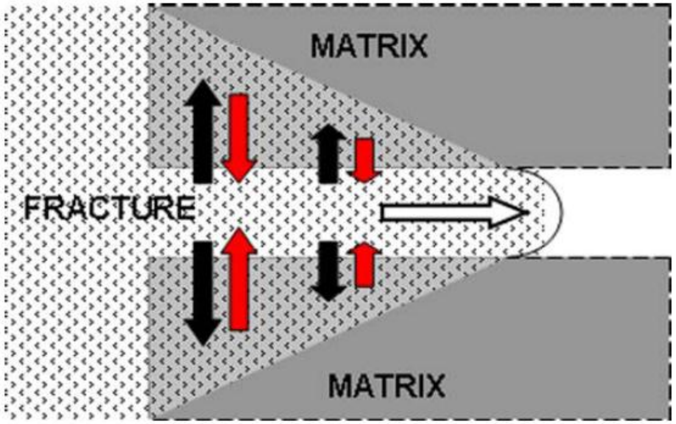


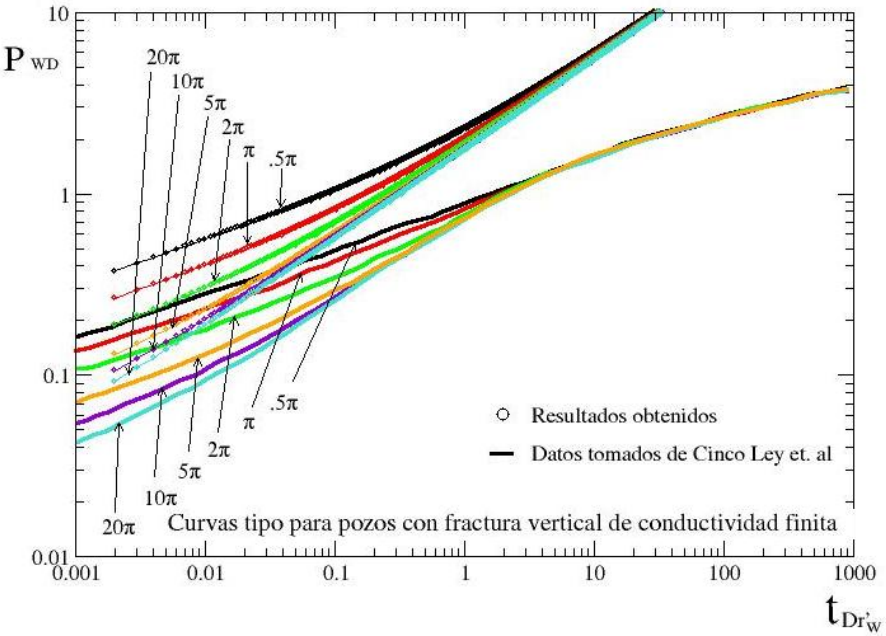
Flujo en Fractura Hidráulica

A. S. Balankin and Dan Silva López (2012)

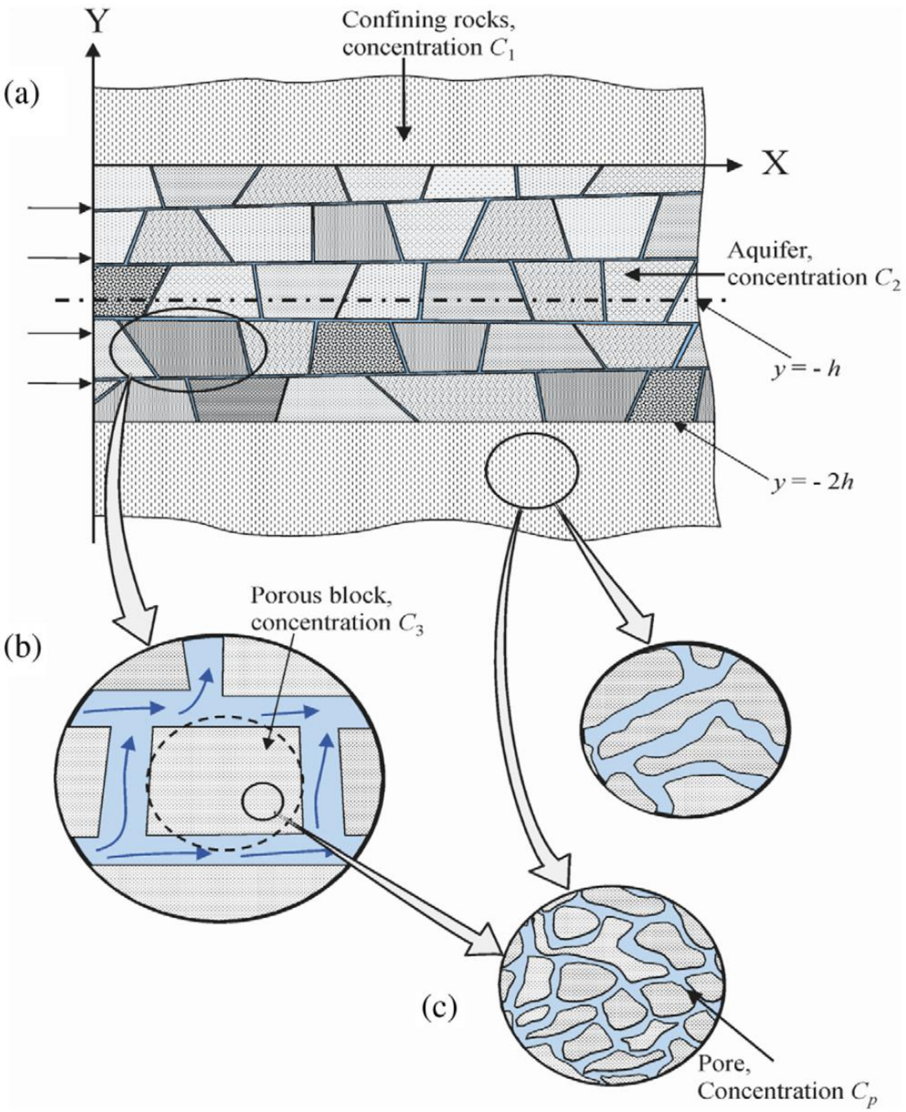


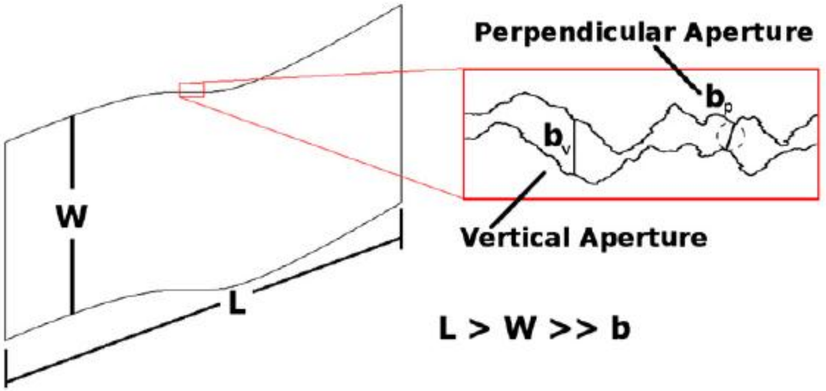


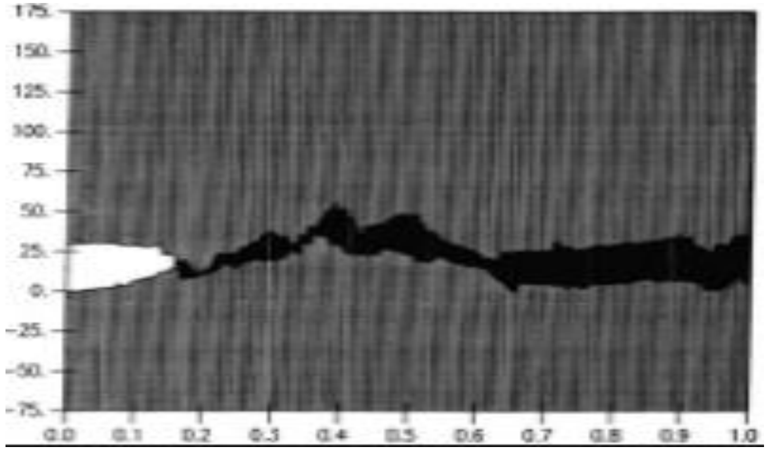




Rugosidad de las Fracturas







Efecto de Rugosidad

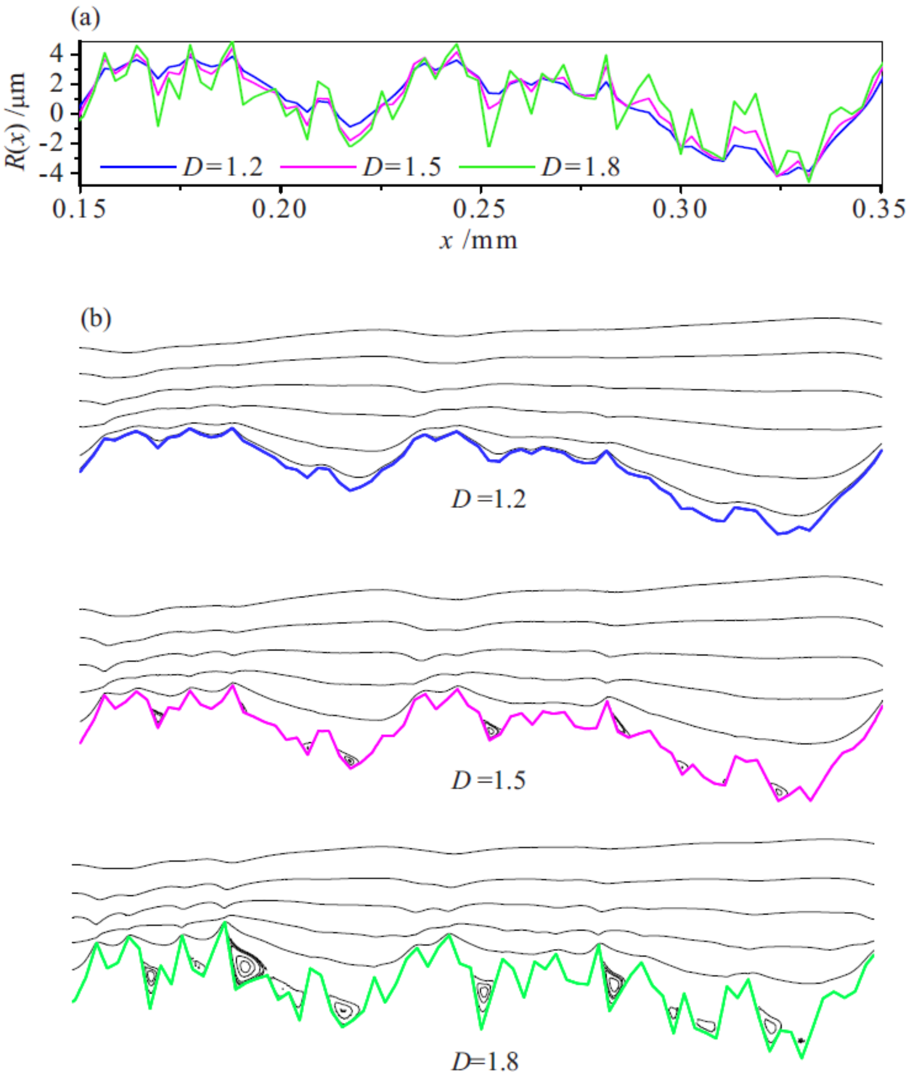


FIG. 1. (Color online) Surface profiles and their roles on the local near-wall streamlines (σ=2 μm): (a) surface profiles and (b) streamlines (D =1.2, 1.5, 1.8).

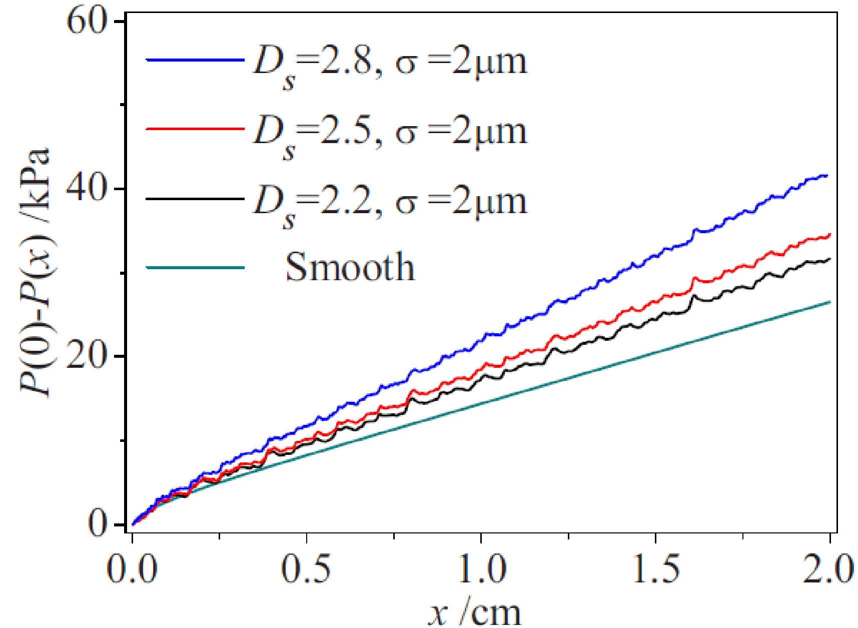
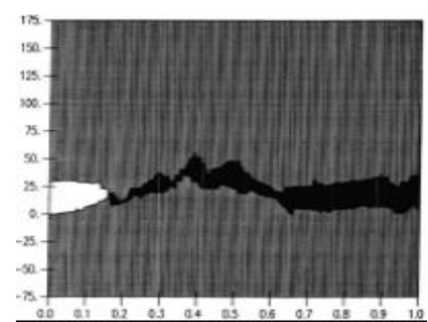
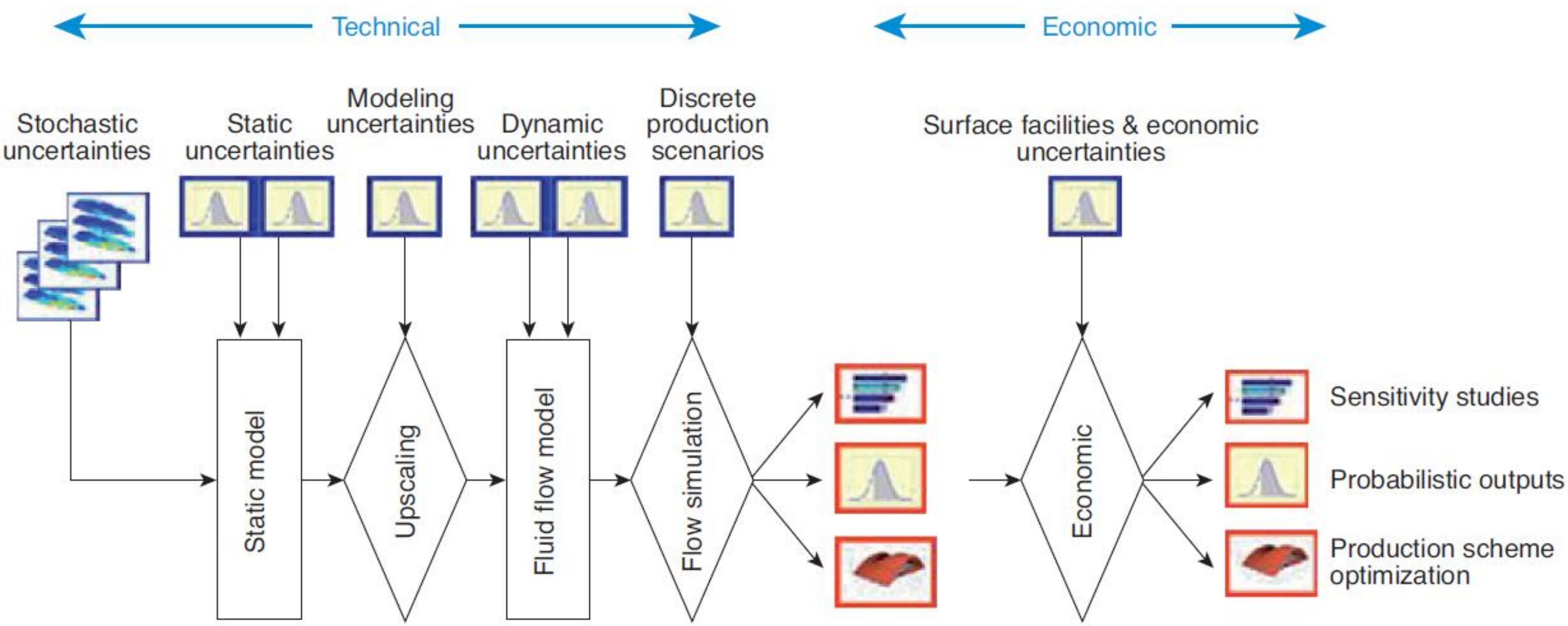


FIG. 2. (Color online) Pressure drop in the rough circular microchannels (Re=1000).



Análisis probabilístico de los resultados de simulación de producción de un pozo



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Electromagnetic fields in fractal continua

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Electrodynamics

ABSTRACT

Fractal continuum electrodynamics is developed on the basis of a model of three-dimensional continuum $\Phi_D^3 \subset E^3$ with a fractal metric. The generalized forms of Maxwell equations are derived employing the local fractional vector calculus related to the Hausdorff derivative. The difference between the fractal continuum electrodynamics based on the fractal metric of continua with Euclidean topology and the electrodynamics in fractional space F^α accounting the fractal topology of continuum with the Euclidean metric is outlined. Some electromagnetic phenomena in fractal media associated with their fractal time and space metrics are discussed.

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Stresses and strains in a deformable fractal medium and in its fractal continuum model

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ABSTRACT

The model of fractal continuum accounting the topological, metric, and dynamic properties of deformable physical fractal medium is suggested. The kinematics of fractal continuum deformation is developed. The corresponding geometric interpretations are provided. The concept of stresses in the fractal continuum is defined. The conservation of linear and angular momentums is established. The mapping of mechanical problems for physical fractal media into the corresponding problems for fractal continuum is discussed.

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Physics in space–time with scale-dependent metrics

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A B S T R A C T

We construct three-dimensional space R^3_γ with the scale-dependent metric and the corresponding Minkowski space–time $M^4_{\gamma,\beta}$ with the scale-dependent fractal (D_H) and spectral (D_S) dimensions. The local derivatives based on scale-dependent metrics are defined and differential vector calculus in R^3_γ is developed. We state that $M^4_{\gamma,\beta}$ provides a unified phenomenological framework for dimensional flow observed in quite different models of quantum gravity. Nevertheless, the main attention is focused on the special case of flat space–time $M^4_{1/3,1}$ with the scale-dependent Cantor-dust-like distribution of admissible states, such that D_H increases from $D_H = 2$ on the scale $\ll \ell_0$ to $D_H = 4$ in the infrared limit $\gg \ell_0$, where ℓ_0 is the characteristic length (e.g. the Planck length, or characteristic size of multi-fractal features in heterogeneous medium), whereas $D_S \equiv 4$ in all scales. Possible applications of approach based on the scale-dependent metric to systems of different nature are briefly discussed.

Gracias por su atención